The Term Structure of Interest Rates in Iceland

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Term Structure of Interest Rates (TSIR)

- A critical issue, not only from a theoretical point of view, but also for all market participants including banks, regulators and financial institutions.
- An essential ingredient in the valuation and hedging of all fixed income securities.
- Necessary for financial planning and for implementing monetary policy.

TSIR and Monetary Policy

- For monetary policy, the structure is of importance in two respects.
  - It contains information not only on market expectations of future interest rate movements but also future developments in inflation and the business cycle.
  - The relationship between short-term and long-term interest rates is relevant to the monetary policy transmission mechanism; although the monetary policy has a crucial impact on the short end of the term structure, it is mainly longer-term interest rates which influence decisions on investment, the acquisition of consumer durables (owner-occupied housing).

Current and Dynamic Approaches

- Current term structure of spot rates
  - Required for valuing and hedging cash flows that are linearly related to the discount function.
  - Spline curve-fitting methods
  - Nelson-Siegel-Svensson: parsimonious representations of the yield curve—limiting the number of parameters and giving more stability to the term structure.
- Dynamics of the term structure of interest rates
  - Required for valuing and hedging cash flows that are non-linear function of the term structure.
  - One (Vasicek, CIR etc.) or multi factor (affine) models.

Arbitrage and Bond Pricing

- The fundamental asset pricing expression:

$$1 = E_t[M(t+1)R(t, t+1)]$$

  - where $M(t+1)$ is the pricing kernel or stochastic discount factor and $R(t+1)$ denotes the one-period return on all traded assets at all dates t.
- The one-period return on $n+1-$period bond is $R(t+1) = P(t+1, n)/P(t, n+1)$, so the bond prices satisfy

$$P(t, n) = E_t[M(t+1)P(t+1, n-1)]$$

- Recursive computation of bond prices with $P(t, 0) = 1$.

One-factor Models

- A single factor (the short rate r) determines bond prices.
- The most popular models constructed by Vasicek (1977) and Cox, Ingersoll, and Ross (1985).
    - Four parameters: three governing the dynamic behavior of the short rate and one controlling the market's valuation of risk.
- With these ingredients, theory then tells us how long rates are connected to the short rate.

Vasicek

- A single state variable z (the short rate r) follows a first-order autoregression:

$$
\begin{aligned}
z_{t+1} &= \psi z_t + (1-\psi)\theta + \sigma\epsilon_{t+1} \\
&= z_t + (1-\psi)(\theta - z_t) + \sigma\epsilon_{t+1}
\end{aligned}
$$

- where
  1. θ = the mean of z
  2. $(\sigma^2/(1-\psi^2))$ σ^2 = the (un)conditional variance
  3. $1-\psi$ = the adjustment speed of z to its long-term mean.
  4. The pricing kernel M:
     $-\log M_{t+1} = \delta + z_t + \lambda\epsilon_{t+1}$; $\lambda =$ the price of risk.

Bond Pricing in the Vasicek Model

- The price of an $n-$period bond

$$P(t,n) = \exp[-A_n - B_n z_t]$$

- The fundamental bond pricing equation is:

$$\log P(t,n) = E_t[\log M(t+1) + \log P(t+1,n-1)] \\ + \frac{1}{2} Var_t[\log M(t+1) + \log P(t+1,n-1)].$$

Bond Pricing in the Vasicek Model

- Substituting $P(t,n)$, $P(t+1,n-1)$ and $M(t+1)$ and using $E_t\left[M(t+1)\right] = \exp[-r_t^f] = \exp[-z_t]$ yields

$$\begin{aligned} A_n + B_n z_t = {} & [A_{n-1} + B_{n-1}(1-\psi)\theta] + [1 + B_{n-1}\psi]z_t \\ & - \tfrac{1}{2}B_{n-1}^2\sigma^2 z_t - B_{n-1}\lambda\sigma z_t. \end{aligned}$$

- Separating the coefficients on the constant and on the terms in z gives us a set of difference equations for A_n and B_n

$$\begin{aligned} B_n &= 1 + (\psi - \lambda\sigma)B_{n-1} - \tfrac{1}{2}B_{n-1}^2\sigma^2, \\ A_n &= A_{n-1} + B_{n-1}(1-\psi)\theta. \end{aligned}$$

The boundary condition $P(t,0) = 1$ implies: $A_0 = B_0 = 0$.

Bond Pricing in the Vasicek Model

- Given values for $\theta, \psi, \sigma, \lambda$ and subject to the above boundary condition we can easily evaluate A_n and B_n.
- Log prices and log yields are linear functions of the interest rate (factor)

$$y(t,n) = -\frac{\log P(t,n)}{n} = \frac{A_n}{n} + \frac{B_n}{n} z_t.$$

One Factor Models

- Vasicek interest rate may actually become negative
- This is not the case in CIR where the random term becomes increasingly smaller as the rate approaches zero

$$z_{t+1} = z_t + (1 - \psi)(\theta - z_t) + \sigma \sqrt{z_t} \epsilon_{t+1}$$

- The time series of the short rate indicates a value of ψ close one, but we need a smaller value to generate the required concavity of the yield curve.
- All yields are linear functions of z. But long yields (yield spread) have higher (lower) autocerrelations than the short rate.
- Innovations in z are conditionally normal but real life innovations have substantial kurtosis.

Multi-factor Vasicek

- k-dimensional vector of state variables:

$$z_{t+1} = \psi_i z_{it} + \sigma_i \epsilon_{it+1}$$

- The pricing kernel M:

$$-\log M_{t+1} = \delta + \sum_i [\lambda_i^2/2 + z_{it} + \lambda_i \epsilon_{it+1}]$$

Multi-factor case

- We modify the pricing procedure explained above for the multi-factor case to obtain:

$$\begin{aligned} B_n(k) &= 1 + [\psi(k) - \lambda(k)\sigma(k)]B_{n-1} - \tfrac{1}{2}B_{n-1}^2(k)\sigma^2(k), \\ A_n &= A_{n-1} + \textstyle\sum_{k=1}^{K} B_{n-1}(k)(1-\psi(k))\theta(k). \end{aligned}$$

  with boundary conditions $A_0(i) = B_0(i) = \cdots B_0(K) = 0$.

- Similarly, in the multi-factor case the yield-to-maturity of a zero-coupon bond in discrete time is defined as

$$y(t,n) = -\frac{\log P(t,n)}{n} = \frac{A_n}{n} + \sum_{k=1}^{K} \frac{B_n(k) z_{kt}}{n}.$$

Estimation of Multi-factor Models

- Deviations of observed prices from model prices: (i) ‘observation error’; (ii) ‘specification error’ and (iii) ‘estimation error’.

$$\tilde{P}(t,n) = P(t,n) + \tilde{e}(t,n)$$

  where $\tilde{P}(t,n)$ is the observed price of bond n at time t, $P(t,n)$ is the model price, and the $e(t,n)$ are independently and identically distributed over bonds with mean zero and constant variance.

- Given that N bonds with different maturities are observed, the N corresponding yields can be stacked to obtain the following representation: $\sum[\tilde{P}(t,n) - P(t,n)]^2$.

Estimation Methods

- State-space models allow for measurement errors in the observed yields to maturity, and is therefore useful for simultaneous estimation using yields on many bonds with different maturities
- To estimate unobservable state variables from a noisy panel-data we use the Kalman filter
- The Kalman filter provides an optimal solution to prediction, updating and evaluating the likelihood function.
- Maximum Likelihood methods

Data and Estimation Problems

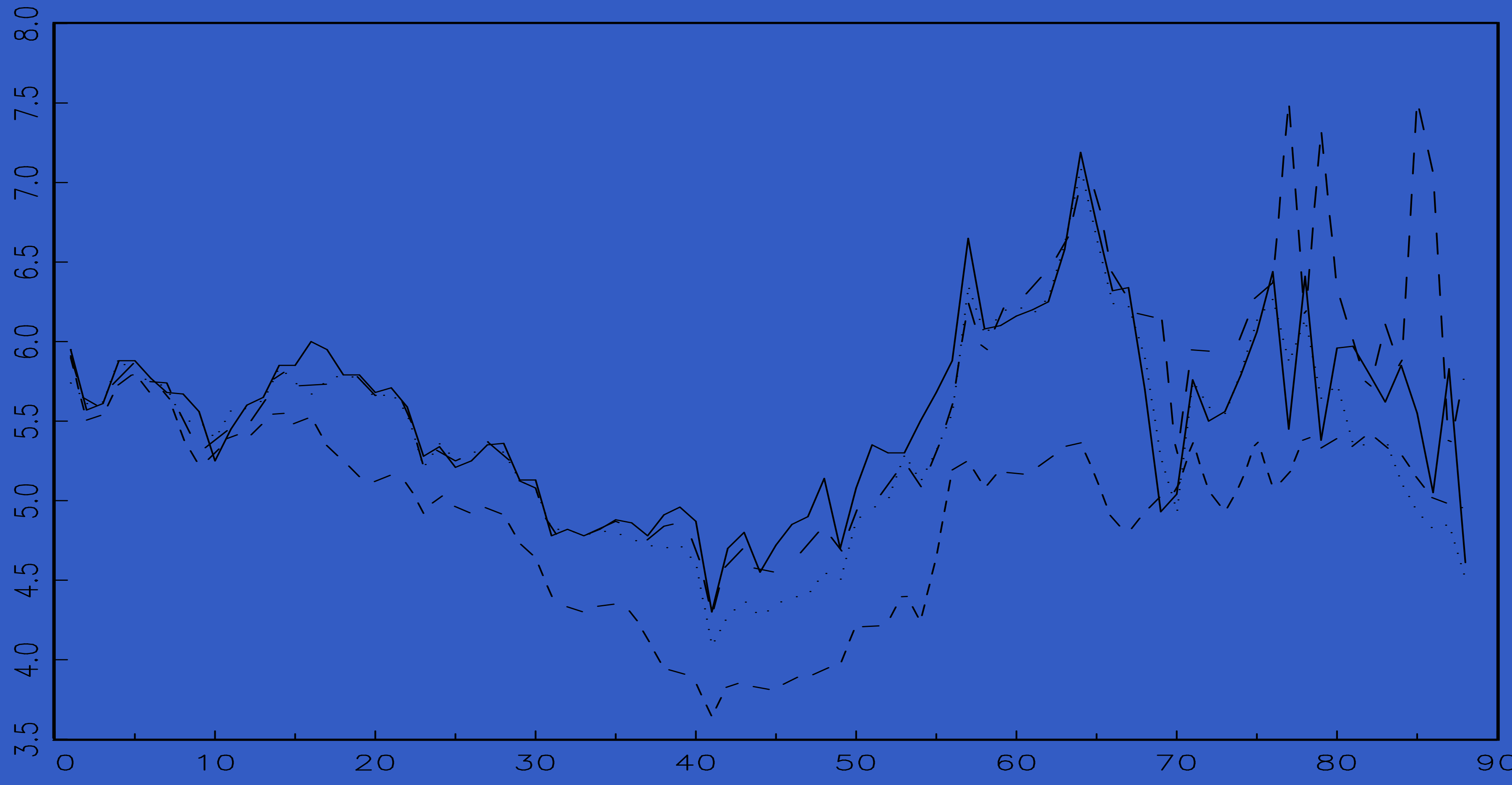
- The Kalman filter requires a complete panel of bond prices (or yields).
- Problem with sparse bond price data: (i) insufficient number of observations and/or (ii) insufficient short and long-term bonds.
- Modify the Kalman filter approach to deal with the missing observation problem.
- Our solution is to use yields and their maturities in a time-varying fashion.

Data

Yields on Four Bonds: Sep 1995 - Dec 2002

RS03-0210/K	RS04-0410/K	RS05-0410/K	RS15-1001/K
5.95	5.91	5.74	5.90
5.57	5.56	5.61	5.50
5.61	5.63	5.62	5.54
5.88	5.80	5.90	5.74
5.88	5.87	5.77	5.80
5.77	5.74	5.76	5.67
5.68	5.74	5.71	5.69
5.67	5.50	5.50	5.39
...	...	...	...
5.83	5.31	4.85	4.98
4.61	5.83	4.50	4.95

Figure: Actual Yields on Four Bonds

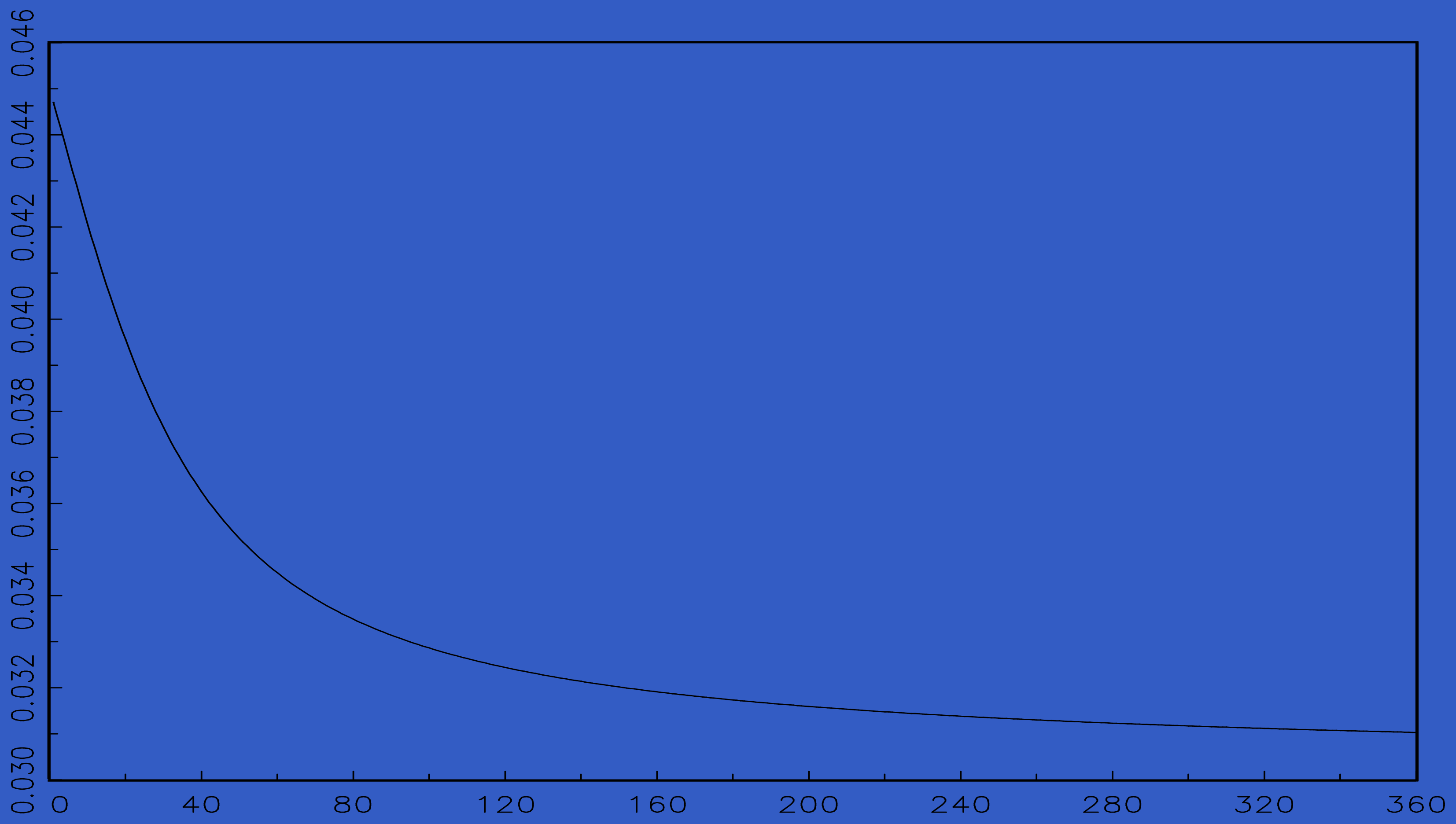


Estimated Values

θ	$1-\psi$	σ	λ
0.049	0.039	0.033	0.0172
(0.0093)	(0.0146)	(0.0058)	(0.0079)
0.047	0.038	0.033	0.0175
(0.0091)	(0.0142)	(0.0055)	(0.0074)
0.044	0.039	0.031	0.0164
(0.0090)	(0.0125)	(0.0063)	(0.0062)
0.040	0.032	0.027	0.0189
(0.0081)	(0.0140)	(0.0027)	(0.0051)

χ^2 stat: 4.2803e+001 and P-Value: 4.0505e-008.

Figure: Estimated CIR Yield Curve



Extensions

- Joint estimation of the current term structure and and its dynamics for markets with infrequent trading.
- Incorporating the bussiness cycle effects on the term structure of interest rates - Markov regime switching term structure of interest rates.
- To combine arbitrage-free models and monetary macroeconomics.

Term Structure with Regime Shifts

- **Direct statistical evidence** on that the interest rate follows a regime switching process
  - Hamilton (1988), Sola and Driffill (1994) and Garcia and Perron (1996).
- **Indirect statistical evidence** in the form of the parameter instability of single-regime interest rate models over the 1979-82 period.
  - Cai (1994), Pearson and Sun(1994) and Brenner (1996)

Term Structure with Regime Shifts

- **Economic reasons**:
  - Business cycle expansion and recession “regimes” have regime switching effects on nominal interest rates.
  - Changes in monetary policy and exchange rate regime have regime effects on nominal interest rates.

Term Structure with Regime Shifts

- The factor process is assumed to be a **mean-reverting square root process** (CIR):

$$z_{t+1} = z_t + (1 - \psi(X))(\theta(X) - z_t) + \sigma(X)\epsilon_{t+1}$$

  where X denotes **discrete states** which are said to be regime 0 and 1.
- The switch between the two regimes is governed by a Markov chain with intensity matrix $H = (h_{ij})$.
- Mixing diffusion processes with point processes (Markov chain) leads to a **Hidden Markov Model**.

Alternative Specifications

- Model 1: Regime switching in volatility

  $$z_{t+1} = z_t + (1-\psi)(\theta - z_t) + \sigma(X)\epsilon_{t+1}$$

- Model 2: RS in volatility and convergency rate

  $$z_{t+1} = z_t + (1-\psi(X))(\theta - z_t) + \sigma(X)\epsilon_{t+1}$$

- Model 3: RS in volatility and long-run rate

  $$z_{t+1} = z_t + (1-\psi)(\theta(X) - z_t) + \sigma(X)\epsilon_{t+1}$$

- Model 4: RS in all parameters

  $$z_{t+1} = z_t + (1-\psi(X))(\theta(X) - z_t) + \sigma(X)\epsilon_{t+1}$$

- Model 5: RS in the market price of factor risk and the inclusion of the market price of regime-switching risk.

Joint Model of Macroeconomics and Term Structure Dynamics

- Difficulties in estimating the term structure of interest rates
- Affine multi-factor models with latent factors do not tell us anything about the determinants of the interest rate level itself.
    - Current monetary policy determines interest rate at the short end.
        - The central bank sets the short-term interest rate in response to inflation, the economic situation and other relevant macroeconomic variables.
    - Long-term interest rates reflect expected economic developments and risk premiums

Joint Model of Macroeconomics and Term Structure Dynamics

- Multi-factors = macroeconomic variables + unobservable factors
- Macroeconomic variables: inflation and business-cycle variable (measured by potential output utilisation)
- Joint dynamics of inflation and output gap is described by a VAR model.
- Arbitrage-free interest rates across all maturities are given as a liner function of inflation, the output and unbservable additional factors.

Joint Model of Macroeconomics and Term Structure Dynamics

- Multi-factors (Z) = macroeconomic variables (Z^m) + unobservable factors (Z^u)

$$Z_{t+1} = \Phi Z_t + \varepsilon_{t+1}$$

- Market price of risk

$$\lambda_{t+1} = d + DZ_{t+1}$$

- A VAR(p) specification for the dynamics of Z^m.
- The latent factors follow a VAR(1) process.

Estimation of the Macro-Term Structure Dynamics Model

1) One-month yields are regressed on the inflation and output variables

$$y(t,1) = a + b_1' Z_t^m + b_2' Z_t^u$$

(2) VAR(p) of Z^m is also estimated using OLS.

(2) The remaining model parameters are determined using a maximum likelihood approach. To do so the model is converted in the state-space form.

The Use of The Macro-Term Structure Dynamics Model

- To gauge the impact of inflation and cyclical fluctuations on current and future interest rates
  - Impulse response analysis. Impacts of a one-off positive inflation shock on interest rates with times to maturity of 3m, 1y, 5y and 10y.
    - The inflation shock increases the current period inflation rate, the resulting tighter monetary policy causes the short-term rate to rise.
    - Inflation persistence leads to the original effect being reduced only gradually, ie inflation remains above its initial level.
    - As a result, all the future short-term interest rates rise as well, albeit to a decreasing extent.
    - At the same time, the assumed inflation impulse can also affect the evolution of the output gap.
- To determine the time profile of risk premiums for various maturities