The great moderation Icelandic style

by

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Abstract

Reduction in the volatility in macroeconomic time series has been documented for a number of countries. This paper documents similar reduction for the Icelandic economy. The paper estimates the timing of the breakpoint and/or a trend in the variance of the series. The paper finds that the reduction in the variance in changes in Gross National Income (GNI) is larger than the reduction in the variance in the changes in GDP, both because of a reduction in the volatility in terms of trade and because of a reduction in the correlation between changes in GDP and changes in terms of trade. The largest contribution to the decline in the volatility in GNI comes though from the reduction in the volatility in GDP.

The paper finds that the volatility in GDP has declined more than the volatility of its components, except export where the decline is greater. The main reason for the decline in the volatility in export is a decline in the volatility in fishing and fish processing. The paper finds that there is a strong relationship between the volatility in export and the volatility in GDP.

*I’m grateful to Thórarinn G. Pétursson for insightful comments.
1. Introduction

In recent years large decreases in the volatility in macroeconomic time series have been observed in a number of countries. This phenomena, which has been called the great moderation, has been documented for the US data (see e.g. McConnell and Perez-Quiros (2000), Blanchard and Simon (2001) and Stock and Watson (2002)) and for data for several other developed countries (see e.g. Stock and Watson (2003) and Summers (2005)).

The purpose of this paper is to see if similar reduction in volatility can be observed in macroeconomic data for Iceland. The paper finds that there is such a decline in the volatility in GDP. As terms of trade shocks have been important for the Icelandic business cycle we also analyse the volatility in GNI and in the terms of trade. We find that there are significant breakpoints in these series and also in the time series for export. Decreases in the volatility can be observed for other variables even if they are in most cases insignificant when tested with available data.

Most studies of the great moderation use quarterly data. This paper is different in that it uses annual data. The reason is that time series of quarterly data are too short for this kind of analysis as they start in the first quarter of 1997. The use of annual rather than quarterly data means that this paper uses much fewer observations on the volatility during each period of time. In some cases the analysis and the tests below are based on rather few observations. One consequence of depending on few observations is that observed differences are less likely to be significant at the usual significance levels.

During recent years considerable efforts have been put into researching reasons for the great moderation in individual countries. In the US the breakpoint is estimated in the early 1980s\(^1\) not long after Paul Volcker took over as chairman of the US Federal Reserve and changed its monetary policy reducing the average rate of inflation from 8.8% during 1973-1982 to the average rate of 3.8% in 1983-1992. This timing of the break in the US data has caused a lot of discussions about the role of (successful) monetary policy and the role of low inflation in bringing about the observed decline in volatility in macroeconomic variables.

\(^1\) McConnell and Perez-Quiros (2000) find that the breakpoint in the US took place in the first quarter of 1984. Blanchard and Simon (2001) find that for the US the decline in volatility is better viewed as a trend decline than as a one-time break. Stock and Watson (2002) test if the change in volatility can be explained as a trend without a break and reject that hypothesis. They locate a break in the second quarter of 1983.
Ensuring low and stable level of inflation is seen today as the primary task of monetary policy. If the monetary authorities have managed this in such a manner that the public expects it to be able to contain inflationary pressures in the future then long-term planning should be easier, the cost of estimating individual prices should be lower as price changes are less frequent and the real values of nominal contracts more predictable. All this should contribute to a decrease in volatility.

After bringing the inflation under control the monetary authority should try to even out fluctuations in real activity in the economy. This is seen as an independent objective besides low and stable inflation. But to some extend it is also a precondition for bringing about low and stable inflation in an economy where a Phillips curve relationship between the output gap (or the unemployment gap) is expected to feed the inflation. Successful monetary policy should therefore bring about smaller fluctuations in real activity.

Stock and Watson (2002) use a VAR model to estimate the role of different factors in explaining the reduction in volatility in the US and find that improved policy explains some 20-30% of the total reduction. Stock and Watson (2003) explain that “(a) although improved monetary policy played a key role in getting inflation under control, it played, at best, a modest role in the great moderation. This conclusion is reinforced by the international evidence” (p. 11). They also conclude that “the empirical evidence suggests that much-half or more-of the great moderation could be temporary, the result of smaller macroeconomic shocks, in particular smaller common international shocks.” (pp. 11-12) Gordon (2005) finds that most of the reduction in volatility in the US is due to “luck” rather than improvements in policy or structural factors. And if it is “luck” rather than policies that have brought about smaller fluctuations in real activity then it is close to hand to speculate if this “luck” also helped monetary authorities to bring about low and stable inflation rather than the other way around as Gordon (2005) does. Smaller fluctuations in real activity obviously makes it easier for monetary authorities to control inflation. Even if it is fair to say that “the consensus supports the “good luck” hypothesis”2 the debate is not over.

Contrary to the case in the US we find a breakpoint in the volatility in GDP in 1972 at the very beginning of a period of high and unstable inflation which lasted

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2 As stated in Giannone et al. (2007), a paper that argues against this hypothesis.
until the early 1990s when the rate of inflation was brought down to an average of 3.7% during the period 1991-2006 from an average of 33.1% during 1972-1990. Because of large terms of trade shocks in the middle of the 1970s we do not find a breakpoint in the volatility in GNI until 1977. We find a breakpoint in the volatility in terms of trade in 1980.

It is to be expected that terms of trade are mostly independent of domestic policies. If we want to associate the decline in the volatility in Icelandic data with some change in policies or in the structure of the Icelandic economy we are left with the task of trying to find some reason for the break in the volatility in GDP which we date in 1972. This point in time is long before the introduction of the quota system in the Icelandic fisheries which took place in 1984 and also long before the various improvements in monetary policies that were introduced during the 1980s and 1990s.

We analyse changes in the volatility of the main components of GDP from the expenditure side and from the production side. The most notable result is that there is a breakpoint in the volatility in export in 1973. Even if this breakpoint is one year later than the breakpoint in GDP Granger Causality tests indicates a causal link from the volatility in exports to the volatility in GDP but not the other way around. The fact that the estimated breakpoint is one year later in the case of export than in the case of GDP indicates that the precision of the breakpoint test in locating the timing of the break is probably low.

Analysis of the volatility of gross factor income at constant prices by industries during the period 1973-2006 shows a very large decrease in the volatility of fishing and fish processing. Its volatility has decreased from being above the volatility in total export to being below the volatility in total export. It seems reasonable that this decrease in the volatility in fishing and fish processing has contributed more to the overall decrease in the volatility in exports than the much discussed increase in diversity of exports from Iceland. We find several breakpoints in the volatility in fishing and fish processing. We date the last one in 1995.

Several events in the late 1960s and early 1970s may have contributed to the decline in the volatility of fishing and fish processing and in export in general in the beginning of the 1970s: The exclusive fishing zone was extended to 50 nautical miles in 1972 and to 200 nautical miles in 1976. At the same time many large scale fishing vessels (trawlers) were added to the fishing fleet, vessels that could operate in worse weather conditions than the smaller vessels and ensure more stable supply of fish.
Before 1970 the herring fishery caused several large booms in the Icelandic economy followed by serious economic difficulties when natural conditions changed the availability of herring or simply its location. One of these booms took place in the mid-1960s followed by a deep crisis in the late 1960s when the herring stocks collapsed under the combined pressure from fishing and adverse natural conditions. In 1969 the aluminium smelter in Straumsvik started its production causing a significant diversification of the mainly fish based export from Iceland.\textsuperscript{3} We will not make any effort to test formally if these factors contributed to the great moderation in Iceland or how much they may have contributed.

Since 1994 the volatility in macroeconomic time series has been lower than before. This could be traced back to better monetary policies with low and stable rate of inflation. Unfortunately, the amount of available data is so small that even considerable changes in volatility are not significant.

This paper is organised so that Section 2 discusses the declining volatility in GDP, Section 3 discusses declining volatility in GNI, Section 4 discusses declining volatility in terms of trade, Section 5 discusses declining volatility in exports and Section 6 discusses declining volatility in other main macroeconomic variables. Section 7 discusses changes in volatility of gross factor income of the main industries. Section 8 concludes.

2. Declining volatility in GDP

Figure 1 shows the absolute value of deviations of changes in GDP from the mean annual growth of GDP during 1945-2007 of 4.1\% (the broken line with uneven dots), 10 years rolling averages of the absolute value of these deviations (the broken line with even dots) and the absolute value of the deviations from the average growth during the respective 10 years (the unbroken line). The points in the figure are placed so that the last year of the period used in the calculations in each case is indicated on the horizontal axis.

\textsuperscript{3} Before 1969 fish products accounted for more than 90\% of the merchandise export. The production of the smelter in Straumsvik was quite large compared to the size of the Icelandic economy at the time. It contributed some 10-15\% of the annual merchandise export during the 1970s. Its weight in net export was somewhat smaller.
It is evident from Figure 1 that the deviations were larger in the 1950s and 1960s than they were later. It is also evident that shortly after 1970 the deviations declined very much. If we were to pick one year as a start year of the period with smaller deviations the figure suggests that we should pick 1972, which is also the year when the line with equally sized dots starts on a downward slope. This line shows that the deviations increased again in the 1990s but most of that increase is an increase in absolute value of deviations from the average growth over the whole period 1945-2007 while the line showing deviations from 10 years average growth (the unbroken line) shows much smaller increase. During the 1990s the growth in the Icelandic economy was much lower than it was during most of the period before 1990 and again during the period after 1997 as shown in Figure 2.

If we divide the sample into two periods, the first one ending in 1971, we find that during the first period the average GDP growth rate was 4.6%, the average absolute values of deviations was 4.8% and the standard deviation was 5.5%, while during the second period the average growth was 3.7%, the average of absolute values of deviations was 2.4% and the standard deviation was 3.0%. Using an F-test to test if the variances of the two growth rates are identical gives a clear rejection with a p-value of 0.000395.
It is customary in the literature to use a univariate time-series model to estimate how much of the variations in the series can be explained by its past history and how much are shocks that are unpredictable with such a model. The idea is to distinguish between those changes in the volatility of the variable that can be traced back to some predictable time-series process for the variable and possible changes in the parameters of this process and those changes that can be traced to unpredictable shocks.

A general univariate time series model for the stationary variable \( y_t \) can be written:

\[
y_t = c + a(L) \cdot y_{t-1} + \epsilon_t
\]

(1)

where \( c \) is a constant, \( \epsilon_t \) is a white noise shock with a standard deviation of \( \sigma_\epsilon \) and \( a(L) = \sum_{j=0}^{n} a_j L^j \) is a polynomial in the lag operator \( L \). In the case where \( n = 0 \) the variance of \( y_t \) is given by \( \sigma_y^2 = \sigma_\epsilon^2 / (1 - a_0^2) \). This expression shows that a decrease in \( \sigma_y \) may come about because \( \sigma_\epsilon \) has decreased while parameter of the univariate model are stable, but also because \( a_0 \) may have decreased while \( \sigma_\epsilon \) has not decreased at all.

Estimation of an AR(2) model for the difference in logarithms of GDP gave the following results:
Table 1

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>0.462</td>
<td>3.65</td>
<td>0.001</td>
</tr>
<tr>
<td>C(2)</td>
<td>-0.205</td>
<td>-1.62</td>
<td>0.111</td>
</tr>
<tr>
<td>C(3)</td>
<td>0.028</td>
<td>3.86</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Jarque-Bera test for normality does not reject the null of normality (p-value 0.47). A CUSUM test of the stability of the parameters of the model didn’t give a reason for rejecting the null of stable parameters. The Quandt-Andrews unknown breakpoint test did not give any reason to reject this null hypothesis either.

A CUSUM of squares test gave clear indications of significant instability in the standard deviation of the residuals from the equation above. Running the regression $|\hat{\epsilon}_t| = c + \nu_t$, where $\hat{\epsilon}_t$ is the estimated residual in the equation above, $\nu_t$ is an iid normally distributed error term and $c$ is a parameter, and using Quandt-Andrews unknown breakpoint test gives a rejection of the null of no breakpoint in $|\hat{\epsilon}_t|$ (p-value, using Hansen’s method, is 0.02). The Wald-statistic has a maximum in 1972 indicating a breakpoint in the standard deviations of changes in GDP at this point in time.4

Estimating the equation $|\hat{\epsilon}_t| = c + \alpha_t D_{1972,t} + \nu_t$, where $D_{1972,t}$ is a dummy taking the value 1 when $t \geq 1972$ and $c$ and $\alpha_t$ are parameters gives marginally better results than the regression $|\hat{\epsilon}_t| = c + \alpha_t \cdot t + \nu_t$, which assumes that the decline in volatility is gradual. In the latter equation $\alpha_1 = -0.000572$ with a standard deviation of 0.000096 (t-value -5.99). The estimation of the encompassing equation, $|\hat{\epsilon}_t| = c + \alpha_1D_{1972,t} + \alpha_2 \cdot t + \nu_t$, gives a p-value for the estimate of $\alpha_1 = 0.060$ but the p-

4 The expression: $\sqrt{0.5\pi \cdot |\epsilon_t|}$ is an unbiased estimator of the standard deviation of $\epsilon_t$ if $\epsilon_t$ is independently and normally distributed. Test of a breakpoint in $|\epsilon_t| = c$ is therefore also a test of a breakpoint in the standard deviation.
value for the estimate $\alpha_2 = 0.136$ when Newey-White autocorrelation and heteroscedasticity consistent standard deviations are used.

To test if there are further breakpoints in the series the procedure above was repeated with data for the period 1972-2006. Statistics from the CUSUM test and the Quandt-Andrews unknown breakpoint test were far from indicating significant instability in the parameters of the univariate equation for $\text{Dlog(GDP)}$. The CUSUM of squares test and the Quandt-Andrews unknown breakpoint test for the equation $|\hat{\epsilon}_t| = c + \nu_t$ didn’t indicate any significant instability in the volatility in the series.

As can be seen from Figure 1 there was a reduction in the volatility in GDP in the beginning of 1990s. The standard deviation of changes in GDP was 3.3% during 1972-1993 but 2.2% during 1994-2007, but with so few observations this reduction in volatility is not significant.

Directly opposite to what is the case in the US, the timing of the breakpoint in the volatility in Icelandic GDP in 1972 coincides with the start of the period of high inflation which lasted until the early 1990s as shown in Figure 3.

Figure 3

![Annual changes in CPI in Iceland and in USA 1960-2006](image)

This result gives support to the view that the reduction in volatility was not caused by an improvement in monetary policies, at least not in domestic monetary policies.\(^5\)

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\(^5\) Figure 3 certainly suggests that there might be some international mechanisms at work that bring about the observed correlation between the rate inflation in Iceland and in the US.
The inflation was brought down to low levels (actually much lower levels than before 1972) in the early 1990s. As mentioned above the variance of changes in GDP was lower during 1994-2007 than it was during 1972-1993 but this reduction is not statistically significant, at least not yet.

3. Declining volatility in GNI

The academic literature on the great moderation focuses on the volatility in GDP rather than in GNI which means that it ignores terms of trade shocks that are an important source of volatility in open economies like Iceland. Figure 2 above gave some indication of changes in the volatility in GNI. Figure 4 below shows the absolute value of deviations of the growth in GNI in individual years from the average over the whole period which was 4.1%.

![Figure 4](image)

Comparison of Figure 4 and Figure 1 reveals that GNI has been more volatile than GDP. Because the decline in the volatility in GDP in 1972 is so apparent in Figure 1 it is a bit surprising that there does not seem to be a breakpoint in the volatility in GNI in 1972. As will become clearer below the reason for this is that the economy was hit by very large terms of trade shocks in the 1970s, the hike in the oil price and large variations in the prices of Icelandic fish in foreign markets.

Even if it is not easy to pin down the exact year Figure 4 shows clearly that volatility in the growth of the GNI in recent years is less than it was earlier. Figure 5 shows this decline in volatility by depicting rolling standard deviations with a window of 10 years.
This figure shows that the volatility in GNI is larger than that in GDP, but the difference declines with time and is very small during the last 15-20 years. This indicates that the volatility in the terms of trade has been declining over time and/or the covariance between changes in GDP and changes in terms of trade has diminished.

Figure 5 also indicates that there might be several breakpoints in the volatility in GNI. We will try to locate these possible breakpoints using the same methodology as above. First the AR(2) equation for Dlog(GNI) was estimated. The results are in Table 2.

| Sample (adjusted): 1948-2007 |
| Equation: DLOG(GNI)=C(1)*DLOG(GNI(-1))+C(2)*DLOG(GNI(-2))+C(3) |
| Coefficient | Std. Error | t-Statistic | Prob. |
| C(1) | 0.524 | 0.126 | 4.156 | 0.000 |
| C(2) | -0.312 | 0.126 | -2.478 | 0.016 |
| C(3) | 0.030 | 0.008 | 3.712 | 0.001 |

R-squared 0.240 Mean dependent var 0.038
Adjusted R-squared 0.213 S.D. dependent var 0.053
S.E. of regression 0.047 Akaike info criterion -3.216
Sum squared resid 0.128 Schwarz criterion -3.110
Log likelihood 99.44 Hannan-Quinn criter. -3.174
F-statistic 9.001 Durbin-Watson stat 2.036
Prob(F-statistic) 0.000

The stability of the parameters of this equation was tested using the CUSUM test and the Quandt-Andrews test. Neither test indicated instability.
The CUSUM of squares test indicated instability in the variance of the error term. We estimated the equation $|\hat{e}_t| = c + \nu_t$, where $\hat{e}_t$ is the estimated residuals from the equation in Table 2 and used the Quandt-Andrews test to test for breakpoints. The test indicates a significant breakpoint (p-value 0.0042) in the volatility in 1977. Efforts to locate further breakpoints were unsuccessful. Figure 4 indicates that there was a reduction in volatility in the 1990s compared to 1977-1990 but this difference is not significant. The tests indicated stability for both parameters and volatility.

Estimating the equation $|\hat{e}_t| = c + \alpha_t \cdot t + \nu_t$ gives a significant value on $\alpha_t = -0.000551$ with a standard deviation of 0.000188 (t-value -2.94) but the result is inferior to the results from estimating the equation $|\hat{e}_t| = c + \alpha_t D_{1977,t} + \nu_t$ where $D_{1977,t}$ is a dummy taking the value 1 when $t \geq 1977$. Estimating the more general model $|\hat{e}_t| = c + \alpha_t D_{1977,t} + \alpha_2 \cdot t + \nu_t$ gives that $\alpha_t$ is significant while $\alpha_2$ is insignificant indicating that the model where there is one breakpoint is significantly better in this case than the model where the reduction in volatility is gradual.

4. Declining volatility in terms of trade

Figure 6 below shows that volatility in terms of trade has decreased.

![Figure 6](image)

Estimating a univariate process for the changes (first differences) in terms of trade (ToT) using OLS gives:
Table 3

Sample (adjusted): 1948-2007

**Equation:** $D(\Delta T\Omega T) = C(1) \times D(\Delta T\Omega T(-1)) + C(2) \times D(\Delta T\Omega T(-2))$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(\Delta T\Omega T(-1))$</td>
<td>0.214</td>
<td>0.123277</td>
<td>1.739</td>
<td>0.087</td>
</tr>
<tr>
<td>$D(\Delta T\Omega T(-2))$</td>
<td>-0.302</td>
<td>0.119279</td>
<td>-2.531</td>
<td>0.014</td>
</tr>
</tbody>
</table>

R-squared 0.125, Mean dependent var 0.001
Adjusted R-squared 0.110, S.D. dependent var 0.047
S.E. of regression 0.044, Akaike info criterion -3.374
Sum squared resid 0.113, Schwarz criterion -3.304
Log likelihood 103.2, Hannan-Quinn criter. -3.346
Durbin-Watson stat 1.985

The CUSUM test and the Quandt-Andrews test indicate parameter stability while the CUSUM of squares test rejects that the variance is stable. Estimating the equation $|\hat{e}_t| = c + \nu_t$, and using the Quandt-Andrews test to locate breakpoints gives a breakpoint in 1980 (p-value = 0.0032). Efforts to locate further breakpoints were unsuccessful and the model $|\hat{e}_t| = c + \alpha \cdot D_{1980_t} + \nu_t$ where $D_{1980_t}$ is a dummy taking to value 1 when $t \geq 1980$ is significantly better than the model $|\hat{e}_t| = c + \alpha \cdot t + \nu_t$, indicating that the reduction in the volatility is more like a one off change than a trend.

It can be seen from Figure 6 above that the decrease in volatility in terms of trade for all goods and services is partly due to a decrease in the volatility in terms of trade for goods but also to the fact that the volatility in terms of trade for services is much lower than the volatility in changes in the terms of trade for goods. In recent years the share of services in total export and import has been increasing. It should though be noted that it is not possible to exclude that the small variance in the changes in terms of trade for services is mainly due to the fact that there are very few direct estimates of changes in the prices of services and Statistics Iceland therefore relies on general price indices for estimating changes in the prices of these services.

Finally we will consider the role of the correlation between changes in GDP and changes in terms of trade. Figure 7 shows how the coefficient of correlation has changed during 1945-2007 by plotting coefficients of correlation based on a moving window of 15 years of data.
The figure shows clearly that the correlation between the two variables has been quite volatile but also that during recent years it has been much lower than previously.

To get some estimates for the contribution of the different factors to the reduction in the volatility in $GNI_t$, let us consider the definition of $GNI_t$:

$$GNI_t = GDP_t + Plrow_t + (PX_t/PM_t - 1) \cdot X_t$$  \hspace{1cm} (2)$$

where $Plrow_t$ is primary incomes receivable from the rest of the world, $PX_t$ is the price of export, $PM_t$ is the price of import and $X_t$ is export in period $t$.

Until very recently the changes in the terms of trade effect dominated the changes in the primary incomes as can be seen from Figure 8.\(^6\) The contribution of the volatility in the primary incomes to the overall volatility in the GNI was therefore negligible.

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\(^6\) The data on important parts of the primary incomes are probably less reliable than most national economic data. It is also relevant to ask if the effects of changes in primary incomes has the same effect as the comparable changes in the terms of trade effect, especially in the short run.
It is possible to show (see Appendix A) that if the variability in the primary incomes is negligible and the changes in the variables are not very large then the following formula is approximately valid:

$$D\log(GNI_t) \approx D\log(GDP_t) + \frac{X_t}{GDP_t} \cdot D\log\left(\frac{PX_t}{PM_t}\right)$$

(3)

It is also possible to show that if the ratio $X_t/GDP_t$ is also stable then the following approximate formula is valid:

$$\sigma_{D\log(GNI)}^2 \approx \sigma_{D\log(GDP)}^2 + \left(\frac{X_t}{GDP_t}\right)^2 \cdot \sigma_{D\log(PX/PM)}^2$$

$$+ 2 \cdot \frac{X_t}{GDP_t} \cdot \sigma_{D\log(GDP)} \cdot \sigma_{D\log(PX/PM)} \cdot \rho_{D\log(GDP),D\log(PX/PM)}$$

(4)

where $\sigma_A$ is the standard deviation of the variable $A$, $\rho_{A,B}$ is the coefficient of correlation between the variables $A$ and $B$ and $\left(\frac{X_t}{GDP_t}\right)$ is a constant estimated as the average of $X_t/GDP_t$.

Table 4 shows the standard deviations of changes in GNI, GDP and ToT for different periods of time. The table also shows the calculated coefficients of correlation between changes in GDP and changes in ToT.
Table 4

<table>
<thead>
<tr>
<th></th>
<th>Standard deviations (%) of relative changes in</th>
<th>Correlation of changes in GDP and ToT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GNI</td>
<td>GDP</td>
</tr>
<tr>
<td>1946-1971</td>
<td>5.5</td>
<td>4.2</td>
</tr>
<tr>
<td>1946-1971</td>
<td>7.0</td>
<td>5.5</td>
</tr>
<tr>
<td>1961-1971</td>
<td>7.0</td>
<td>5.8</td>
</tr>
<tr>
<td>1981-2007</td>
<td>3.6</td>
<td>3.0</td>
</tr>
<tr>
<td>1994-2007</td>
<td>2.6</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Formula (4) can be used to estimate the marginal contribution of the different variables to the decline in the standard deviations of changes in GNI. If we only let the standard deviation of changes in GDP decline as it did in 1981-2007 and in 1994-2007 compared to 1961-1971, while keeping the standard deviation of ToT and the coefficient of correlation unchanged, then the standard deviation of changes in GNI would have declined to 4.2 in 1981-2007 and to 3.5 in 1995-2007. Allowing only the standard deviation of changes in terms of trade to change would have brought the standard deviation of changes in GNI to 6.7 in both periods, while allowing only the coefficient of correlation to change would have brought the standard deviation of changes in GNI to 7.0 in 1981-2007 (i.e. no decline) and to 6.8 in 1995-2007. This shows that even if a decline in the variance of ToT and a decline in the correlation between changes in GDP and ToT did contribute to the decline in the variance of changes in GNI the overwhelmingly largest contribution came from the decline in the variance of changes in GDP.

5. Declining volatility in export

It has been generally accepted among economist in Iceland that there has been a close statistical and causal link between the conditions in the main export industries in Iceland and the Icelandic business cycle. Fish used to be the main export item and then this meant a close relationship between the shocks hitting the fishing sector and the Icelandic business cycle. A number of researchers have documented this connection. More recent work has shown that this connection between the fishing

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8 See Danielsson (2004).
sector and GDP has almost disappeared in recent year. The correlations between changes in export, terms of trade and GDP continues though to be high.

Figure 9 shows rolling 10 years standard deviations of changes in export and in GDP. This figure indicates that there is a close relationship between the volatility in exports and the volatility in GDP.

The figure shows well how much the standard deviation of changes in export has declined and also how the decline in the standard deviation of change in GDP follows the decline in the standard deviation of changes in export.

There is not much use in estimating a univariate process for changes in export from Iceland. The process of locating possible breakpoints is therefore reduced to estimating the mean of the changes during some period of time and then perform tests on the equation \(|\hat{e}_i| = c + \nu_i\), where \(\hat{e}_i\) is the deviation of the change from the mean. We choose to use the period from 1960-2007. The mean is 0.0453 and the distribution of the deviations from the mean can be considered normal. (p-value of Jarque-Bera test 0.608) The Quandt-Andrews test for the equation \(|\hat{e}_i| = c + \nu_i\) gave a breakpoint in 1973 (p-value for the Maximum Wald F-statistic 0.0741).

Even if the estimated breakpoint in the volatility in export is one year later than the estimated breakpoint in the volatility in GDP Granger Causality test indicates strongly that it is the volatility in export that influences volatility in GDP and not the other way around. The details of the test are in Table 5.
The effects of changes in the volatility in terms of trade on volatility in GDP does not show up in a Granger Causality test but the variable appears as significant explanatory variable in regressions where the volatility in GDP or in GNI are on the left hand side as shown in Tables 6 and 7 below.

### Table 6

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abs((\text{Dlog(E)}))</td>
<td>0.106</td>
<td>0.070</td>
<td>1.509</td>
<td>0.139</td>
</tr>
<tr>
<td>Abs((\text{Dlog(E(-1))}))</td>
<td>0.283</td>
<td>0.066</td>
<td>4.278</td>
<td>0.000</td>
</tr>
<tr>
<td>Abs((\text{Dlog(E(-2))}))</td>
<td>0.134</td>
<td>0.077</td>
<td>1.735</td>
<td>0.090</td>
</tr>
<tr>
<td>Abs((\text{Dlog(ToT(-1))}))</td>
<td>0.278</td>
<td>0.110</td>
<td>2.529</td>
<td>0.015</td>
</tr>
</tbody>
</table>

R-squared 0.398
Adjusted R-squared 0.357
S.E. of regression 0.024
Durbin-Watson stat 1.676

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abs((\text{Dlog(Exp)}))</td>
<td>0.182</td>
<td>0.074</td>
<td>2.430</td>
<td>0.019</td>
</tr>
<tr>
<td>Abs((\text{Dlog(Exp(-1))}))</td>
<td>0.317</td>
<td>0.084</td>
<td>3.770</td>
<td>0.001</td>
</tr>
<tr>
<td>Abs((\text{Dlog(ToT)}))</td>
<td>0.235</td>
<td>0.135</td>
<td>1.747</td>
<td>0.088</td>
</tr>
<tr>
<td>Abs((\text{Dlog(ToT(-1))}))</td>
<td>0.292</td>
<td>0.129</td>
<td>2.274</td>
<td>0.028</td>
</tr>
</tbody>
</table>

R-squared 0.463
Adjusted R-squared 0.426
S.E. of regression 0.028
Durbin-Watson stat 1.652

6. Changes in the volatility in other macroeconomic aggregates

Using the methodology above to search for breakpoints or significant trends in the volatility in the components of the GDP from the expenditure side: private consumption, public consumption, investment and imports gives some results but not as clear breakpoints as in GDP, GNI and exports discussed above. If the whole sample 1945-2007 is used the Quandt-Andrews test gives a breakpoint in private consumption
in 1978 with a p-value 9.1%. The simple model with linear trend and the model with a breakpoint in 1978 give a significant coefficients for these variables. The significance of the dummy for a break in 1978 is much higher than the significance of the linear trend in the model encompassing both variables.

There is a significant breakpoint in the volatility in public consumption in 1986 (p-value 0.050). The test indicates a breakpoint in 1956 in the volatility in imports (p-value 0.043). Efforts to locate a breakpoint after 1956 were unsuccessful. No breakpoint was detected in the volatility in investments. We didn’t find further significant breakpoints or trends to in the data even if the volatility in all macroeconomic aggregates decreases over time as shown in Table 8 below.

Table 8

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>Private cons.</th>
<th>Public cons.</th>
<th>Investm.</th>
<th>Export</th>
<th>Import</th>
</tr>
</thead>
<tbody>
<tr>
<td>1946-2007</td>
<td>4.2</td>
<td>6.5</td>
<td>3.3</td>
<td>17.2</td>
<td>9.4</td>
<td>13.0</td>
</tr>
<tr>
<td>1946-1971</td>
<td>5.5</td>
<td>7.5</td>
<td>3.8</td>
<td>21.3</td>
<td>13.0</td>
<td>15.5</td>
</tr>
<tr>
<td>1961-1971</td>
<td>5.8</td>
<td>8.0</td>
<td>2.2</td>
<td>20.6</td>
<td>10.8</td>
<td>14.3</td>
</tr>
<tr>
<td>1981-2007</td>
<td>3.0</td>
<td>5.7</td>
<td>2.0</td>
<td>15.0</td>
<td>5.3</td>
<td>11.3</td>
</tr>
<tr>
<td>1994-2007</td>
<td>2.2</td>
<td>4.7</td>
<td>1.3</td>
<td>16.7</td>
<td>4.3</td>
<td>11.2</td>
</tr>
</tbody>
</table>

**Relative changes compared to the 1961-1971 period**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative changes</td>
<td>-48.5%</td>
<td>-61.2%</td>
</tr>
<tr>
<td>Private cons.</td>
<td>-29.3%</td>
<td>-42.1%</td>
</tr>
<tr>
<td>Public cons.</td>
<td>-10.4%</td>
<td>-42.7%</td>
</tr>
<tr>
<td>Investm.</td>
<td>-27.5%</td>
<td>-18.9%</td>
</tr>
<tr>
<td>Export</td>
<td>-51.3%</td>
<td>-60.1%</td>
</tr>
<tr>
<td>Import</td>
<td>-21.0%</td>
<td>-21.7%</td>
</tr>
</tbody>
</table>

The definition of $GDP_t$ gives that:

$$GDP_t = C_t + G_t + I_t + X_t - M_t$$

(5)

where $C_t$ is private consumption, $G_t$ is public consumption, $I_t$ is investments, $X_t$ is export and $M_t$ is imports in period $t$.

Using the same methods and similar assumptions as are used in Appendix A to derive equation (4) it is possible to show that the following equation is approximately valid:
\[
\sigma^2_{D \log(Y_t)} = \left( \frac{C}{Y} \right)^2 \sigma^2_{D \log(C_t)} + \left( \frac{G}{Y} \right)^2 \sigma^2_{D \log(G_t)} + \left( \frac{I}{Y} \right)^2 \sigma^2_{D \log(I_t)} + \left( \frac{X}{Y} \right)^2 \sigma^2_{D \log(X_t)} \\
+ \left( \frac{M}{Y} \right)^2 \sigma^2_{D \log(M_t)} + 2 \sum_{A,B} \left( \frac{A}{Y} \right) \cdot \left( \frac{B}{Y} \right) \cdot \sigma_{D \log(A_t)} \sigma_{D \log(B_t)} \rho_{D \log(A_t),D \log(B_t)} \text{ for } A, \ B = C, \ G, \ I, \ X, \ -M \text{ and } A \neq B \tag{6}
\]

In the Icelandic data there are large deviations from the assumptions behind the derivation of equation (6), both the assumptions that relative changes in the variables are small and the assumption that the ratios are stable. In spite of this equation (6) gives some insights into the relationships between volatility in GDP and volatilities in and weights of its components.

Using equation (6) to estimate the marginal contribution of changes in volatility in individual components reveals that the largest contribution comes from the reduction of volatility in private consumption. As discussed above the breakpoint in 1978 is not far from being significant at the 5% level but the importance of the reduction in the volatility in private consumption for the reduction in volatility in GDP comes also from the large weight of private consumption in GDP. Plugging the standard deviation of private consumption in 1981-2007 into equation (6) while keeping the values on all other standard deviations and correlations as they were in 1961-1971 gives 21.2% decline in the standard deviation of changes in GDP. Proceeding similarly with the standard deviation of private consumption in 1995-2007 while leaving all other factors in equation (6) as they were in 1961-1971 the standard deviation of changes in GDP declines by 31.6%. Table 9 shows the results of this kind of exercises for all components of GDP.

<table>
<thead>
<tr>
<th>Table 9</th>
<th>Marginal contribution to the reduction in St.dev.(Dlog(GDP)) from:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Private cons.</td>
</tr>
<tr>
<td>1981-2007</td>
<td>-21.2%</td>
</tr>
<tr>
<td>1994-2007</td>
<td>-31.6%</td>
</tr>
</tbody>
</table>

Table 9 shows that the reduction in the volatility in public consumption has a very small impact on the volatility in GDP when equation (6) is used. It should be noted though that the increase in the share of public consumption from 15% in 1961-1971 to
23% in 1981-2007 and to 24% in 1995-2007 had a larger impact on the volatility in GDP because the volatility in public consumption is much smaller than the volatility in other components of GDP. Equation (6) is based on approximations which ignore this kind of effects.

Table 9 also shows that the marginal effect of a reduction in volatility in imports is actually an increase in volatility in GDP. The reason is that there are the coefficient of correlation between changes in private consumption and changes in imports is very high and also the coefficient of correlation between changes in investments and changes in imports.

We noted above that volatility in GNI declined faster than volatility in GDP (see Figure 5). We can now add that the volatility in GDP has declined faster than the volatility in most other variables shown in Table 8. The only exception is export where the reduction in volatility is possibly greater. Figure 10 shows the ratio of the standard deviations of changes in private consumption and the standard deviation of changes in GDP on the one hand (the broken line) and the standard deviation of changes in GNI (the unbroken line). In both cases the figure shows this ratio calculated for a rolling window of 15 years.

The figure shows that volatility in consumption has increased relative to volatility in GDP and especially relative to volatility in GNI. If the objective of economic policy and efficient financial markets is to create conditions for maximisation the discounted value of utility which is a concave function of consumption (cf. Lucas, 1987), i.e. to create conditions for smoothing of consumption over time, then Figure
10 indicates that economic policy and financial markets in Iceland have been doing a poor job. Not only is the volatility in private consumption much higher than in GDP and in GNI, there is also an increasing trend in the ratios of standard deviations of changes in these variables indicating that the performance of economic policy and financial markets has been deteriorating over time.

That volatility in the aggregates, GDP and GNI, has decreased much more than the volatility in their components is also relevant for forecasting of these variables. The decrease in volatility in GDP and GNI makes forecasting of changes in these variables easier using naive univariate methods that do not make use of economic theory. The usual macroeconomic forecasting of changes in GDP and GNI based on forecasting of their components and then adding, subtracting and multiplying in accordance with equations (5) and (2) above have to confront the smaller reduction in the volatility in the components. When using equations (5) and (2) the errors in the forecasting of the components are added up in the forecast errors of the aggregates. It is reasonable to expect that the forecasting accuracy of the latter method improves less in these circumstances than that of the univariate methods when forecasting changes in GDP and GNI.

7. Reduction in the volatility by industries

Unfortunately we do not have data on the volatility in the individual industries before 1973. This means that in most cases we are not able to study the role of individual industries in the breaks in volatility in GDP and in export that occurred in the beginning of the 1970s. It is though worth studying changes in the volatility in individual industries since 1973 using the national accounts from the production side from Statistics Iceland. These data show a large reduction in the volatility in fishing and fish processing and there has also been considerable reduction in the weight of these traditionally very volatile industries in the Iceland economy.

Table 7.1 shows the changes in volatility by industries using the standard deviations of the changes in gross factor income during 15 years as a measure of volatility.
Table 10

<table>
<thead>
<tr>
<th>Standard deviation of changes (%) in gross factor income by industries</th>
<th>1974-88</th>
<th>1984-98</th>
<th>1992-06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, hunting and forestry</td>
<td>10.1</td>
<td>3.7</td>
<td>3.0</td>
</tr>
<tr>
<td>Fishing, operation of fish hatcheries and fish farms</td>
<td>13.0</td>
<td>9.7</td>
<td>3.7</td>
</tr>
<tr>
<td>Processing and preserving of fish and fish products</td>
<td>8.9</td>
<td>5.8</td>
<td>6.4</td>
</tr>
<tr>
<td>Fishing and fish processing</td>
<td>9.8</td>
<td>6.6</td>
<td>3.9</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>6.3</td>
<td>5.2</td>
<td>3.3</td>
</tr>
<tr>
<td>Electricity, gas and water supply</td>
<td>3.3</td>
<td>1.9</td>
<td>3.7</td>
</tr>
<tr>
<td>Construction</td>
<td>7.4</td>
<td>6.2</td>
<td>10.1</td>
</tr>
<tr>
<td>Wholesale and retail trade; repair of vehicles&amp;househ. goods</td>
<td>5.6</td>
<td>5.5</td>
<td>5.1</td>
</tr>
<tr>
<td>Hotels and restaurants</td>
<td>8.1</td>
<td>7.1</td>
<td>7.0</td>
</tr>
<tr>
<td>Transport, storage and communication</td>
<td>4.7</td>
<td>5.3</td>
<td>5.2</td>
</tr>
<tr>
<td>Financial, real-estate, renting and business activities</td>
<td>1.3</td>
<td>3.1</td>
<td>4.3</td>
</tr>
<tr>
<td>Other service activities</td>
<td>2.4</td>
<td>2.6</td>
<td>2.4</td>
</tr>
<tr>
<td>Total</td>
<td>2.6</td>
<td>3.3</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Data on merchandise export show that volatility in fishing was larger before 1973 than after 1973 but Table 10 shows that this volatile industry was still the most volatile industry during the period from 1973-1988. Agriculture is in the second place and fish processing in the third place. The table shows that the volatility in these industries decreases very much between the periods 1973-1988 and 1991-2006.

Note that data on overall volatility in GDP in Table 10 do not indicate a reduction in volatility during this period. This means that increased volatility in those industries where the volatility increased compensated for the reduction in volatility in fishing, fish processing and agriculture.

Table 7.2 shows that the weight of these industries in the total gross factor income in the economy has also decline very much.

Table 11

<table>
<thead>
<tr>
<th>Share (%) of main industries in total gross factor income</th>
<th>1973</th>
<th>1990</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, hunting and forestry</td>
<td>5.2</td>
<td>2.5</td>
<td>1.4</td>
</tr>
<tr>
<td>Fishing, operation of fish hatcheries and fish farms</td>
<td>7.2</td>
<td>9.6</td>
<td>4.7</td>
</tr>
<tr>
<td>Processing and preserving of fish and fish products</td>
<td>8.2</td>
<td>4.8</td>
<td>2.0</td>
</tr>
<tr>
<td>Fishing and fish processing</td>
<td>15.4</td>
<td>14.4</td>
<td>6.7</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>20.9</td>
<td>16.8</td>
<td>11.5</td>
</tr>
<tr>
<td>Electricity, gas and water supply</td>
<td>2.9</td>
<td>4.1</td>
<td>4.0</td>
</tr>
<tr>
<td>Construction</td>
<td>12.0</td>
<td>7.9</td>
<td>10.6</td>
</tr>
<tr>
<td>Wholesale and retail trade; repair of vehicles&amp;househ. goods</td>
<td>10.6</td>
<td>11.8</td>
<td>10.6</td>
</tr>
<tr>
<td>Hotels and restaurants</td>
<td>1.2</td>
<td>2.2</td>
<td>1.6</td>
</tr>
<tr>
<td>Transport, storage and communication</td>
<td>9.3</td>
<td>8.0</td>
<td>6.4</td>
</tr>
<tr>
<td>Financial, real-estate, renting and business activities</td>
<td>15.3</td>
<td>17.7</td>
<td>26.9</td>
</tr>
<tr>
<td>Other service activities</td>
<td>14.5</td>
<td>19.3</td>
<td>22.1</td>
</tr>
<tr>
<td>Total</td>
<td>99.3</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>
Figure 11 shows the decline in the volatility of some of the industries included in Table 10 in greater detail than is possible in a table. Rolling windows of 15 years are used.

**Figure 11**

15 years rolling standard deviations of changes in the volume indices of gross factor income by industries 1973-2006

By linking the information on fishing and fish processing from the production accounts with other information it is possible to extend the series further back. Figure 12 shows absolute values of changes in the production of fishing and fish processing and rolling 10 and 15 years standard deviations for these changes for the period 1945-2006. For comparison purposes absolute values of changes in the volume of total export has been included. The figure shows that the volatility in fishing and fish processing has changed from being above the volatility in total export to actually become lower than the volatility in total export.

Testing for breakpoints in the volatility results in several breakpoints. The first breakpoint is in 1957 (p-value 0.0041) and the second one in 1987 (p-value 0.0016). Using the Quandt-Andrews unknown breakpoint test to test for breakpoints after 1987 using the sample from 1987 does not detect a significant breakpoint but using the test to test for breakpoints in the sample from 1973 gives a breakpoint in 1995 (p-value of 0.038).
It is reasonable to expect that this large reduction in the volatility in fishing and fish processing has been very important for the success of economic policies in Iceland during the last two decades. To some extent the reduction in this volatility may also be the consequence of economic policies. Fishing and fish processing used to be the main source of instability in the Icelandic economy but now it is less volatile than other export industries. The standard deviation of changes in the gross factor income in fishing and fish processing was 3.9% during 1992-2006 as can be seen from Table 10. During this same period the standard deviation of the changes in total export was 4.3%, of changes in merchandise export was 4.9% and of changes in service export was 8.6%.

8. Conclusions

We have shown above that there is conclusive evidence for the existence of great moderation in volatility in macroeconomic time series for the Icelandic economy. It seems plausible that this moderation came about because of decreases in volatility in export and in terms of trade. The timing of the breakpoints in volatility in export and volatility in GDP in 1972 and 1973 supports this hypothesis. Results from Granger’s causality tests support it also.

The link between changes in volatility in GDP and in export makes it probable that the reasons for the great moderation in Iceland were changes in the conditions of the export industries. Possible domestic factors affecting the volatility of export are
the extension of the exclusive fishing zone in 1972 and in 1976, investments in larger vessels that could ensure stable supply of groundfish, especially cod, decline in the dependence of the very volatile herring catches and also greater diversification of export when the aluminium smelter in Straumsvík started its production in 1969. Later jumps in diversification of export through further investments in energy-intensive industries and gradual increase in diversification of export through more varied production of fish products may have contributed to some further reduction in the volatility in exports.

Even if diversification of the Icelandic exports away from fish seems a plausible explanation for the reduction in volatility in exports the data presented in section 7 above seem to indicate that large reductions in the volatility in fishing and in fish processing itself is responsible for a very large part of this reduction. It was shown that the volatility in fishing and fish processing changed from being very high compared to the volatility in other industries in the first post-war decades and even so late as in 1973-1988 to becoming low compared to other industries in the period 1992-2006.

If large parts of the international decline in macroeconomic volatility is temporary, as suggested by Stock and Watson (2003), it is to be expected that changes in international volatility will be felt in the Icelandic economy through changes in the volatility in export and in terms of trade. In so far as these changes in volatility in exports and terms of trade will come through increases in the unpredictable changes in these variables they will make monetary policy more difficult than it presently is.

Compared to other countries the timing of the great moderation in volatility in GDP in Iceland is quite early. According to Summer (2005, Table 1) the earliest breakpoint in GDP volatility among the G-7 countries plus Australia occurred in Germany in the third quarter of 1971, the second occurred in Japan in the second quarter of 1975 and the third in France in the third quarter of 1976. In the other five countries the breakpoints are found in the 1980s.

According to figures in Summer (2005) the reduction in the volatility measured by the ratio of the variances after and before the breakpoint was larger in Iceland than in the countries discussed in that paper. In Iceland this ratio of the variances before and after 1972 was 30%. The lowest ratio among the G-7 plus Australia was in Australia where it was 46%, followed by Italy and the US where it was 51% and then UK where it was 52%. The highest ratio was in Japan where it was 63%.
In Table 1, p. 15, Stock and Watson (2003) give standard deviations of four quarter changes in quarterly GDP in G-7 countries in 1960-1983 and in 1984-2002. The arithmetic mean for these large economies was 2.6% in the first period and 1.7% in the second. For the Icelandic economy the standard deviations of annual changes are 4.4% for the first period and 2.9% for the second. On this measure the volatility in the Icelandic economy has remained roughly 70% above the volatility in these large economies in both periods. As discussed above the volatility in GDP has declined further in recent years and the standard deviation for the period 1994-2007 is 2.2%, which is lower than the average for the G-7 economies during the period 1960-1983.

We noted above that the decline in volatility in GDP was larger than the decline in volatility in its components, except export. We also noted that the decline in volatility in GNI was larger than the decline in the volatility in GDP.

Appendix A

$GNI_t$ is defined as:

$$GNI_t = GDP_t + Plrow_t + (PX_t / PM_t - 1) \cdot X_t$$  \hspace{1cm} (A.1)

Dividing through with $GDP_t$ gives:

$$\frac{GNI_t}{GDP_t} - 1 = \frac{Plrow_t}{GDP_t} + \left( \frac{PX_t}{PM_t} - 1 \right) \cdot \frac{X_t}{GDP_t}$$ \hspace{1cm} (A.1')

Using that $\log(1 + x) \approx x$ if $x$ is small on $\frac{GNI_t}{GDP_t} - 1$ gives that

$$\log(GNI_t) \approx \log(GDP_t) + \frac{Plrow_t}{GDP_t} + \left( \frac{PX_t}{PM_t} - 1 \right) \cdot \frac{X_t}{GDP_t}$$ \hspace{1cm} (A.2)

Taking the difference and using that Figure 8 above shows that $\frac{Plrow_t}{GDP_t} - \frac{Plrow_{t-1}}{GDP_{t-1}}$ was small before 2000 gives that:

$$D \log(GNI_t) \approx D \log(GDP_t) + \left( \frac{PX_t}{PM_t} - \frac{PX_{t-1}}{PM_{t-1}} \right) \cdot \frac{X_t}{GDP_t}$$

$$+ \left( \frac{PX_{t-1}}{PM_{t-1}} - 1 \right) \left( \frac{X_t}{GDP_t} - \frac{X_{t-1}}{GDP_{t-1}} \right)$$ \hspace{1cm} (A.3)
If the last term on the right hand side is small compared to the other two, which is reasonable in most practical cases, equation (A.3) can be simplified to:

\[ D \log(\text{GNI}_t) \approx D \log(\text{GDP}_t) + \left( \frac{\text{PX}_t}{\text{PM}_t} - \frac{\text{PX}_{t-1}}{\text{PM}_{t-1}} \right) \cdot \frac{X_t}{\text{GDP}_t} \]  

\[ \text{(A.3')} \]

or, if the approximation \( \log(1+x) \approx x \) when \( x \) is small is used on \( \text{PX}_t/\text{PM}_t - 1 \) to:

\[ D \log(\text{GNI}_t) \approx D \log(\text{GDP}_t) + \frac{X_t}{\text{GDP}_t} \cdot D \log \left( \frac{\text{PX}_t}{\text{PM}_t} \right) \]  

\[ \text{(A.3'')} \]

If the variations in the ratio \( X_t/\text{GDP}_t \) are small so that

\[ D \log(\text{GNI}_t) \approx D \log(\text{GDP}_t) + \left( \frac{X_t}{\text{GDP}_t} \right) \cdot \left( \frac{\text{PX}_t}{\text{PM}_t} - \frac{\text{PX}_{t-1}}{\text{PM}_{t-1}} \right) \]

\[ + \left( \frac{X_t}{\text{GDP}_t} - \frac{X_t}{\text{GDP}_t} \right) \cdot \left( \frac{\text{PX}_t}{\text{PM}_t} - \frac{\text{PX}_{t-1}}{\text{PM}_{t-1}} \right) \]

\[ = D \log(\text{GDP}_t) + \left( \frac{X_t}{\text{GDP}_t} \right) \cdot \left( \frac{\text{PX}_t}{\text{PM}_t} - \frac{\text{PX}_{t-1}}{\text{PM}_{t-1}} \right) \]  

\[ \text{(A.4)} \]

where \( (X_t/\text{GDP}_t) \) is a constant (estimated as the average of \( X_t/\text{GDP}_t \)) then:

\[ \text{Var}[D \log(\text{GNI}_t)] \approx \text{Var}[D \log(\text{GDP}_t)] + \left( \frac{X_t}{\text{GDP}_t} \right)^2 \cdot \text{Var}[D \log \left( \frac{\text{PX}_t}{\text{PM}_t} \right)] \]

\[ + 2 \cdot \left( \frac{X_t}{\text{GDP}_t} \right) \cdot \text{Cov}[D \log \left( \frac{\text{PX}_t}{\text{PM}_t} \right), D \log(\text{GDP}_t)] \]  

\[ \text{(A.5)} \]

Noting that \( \text{Cov}(A,B) = \sigma_A \cdot \sigma_B \cdot \rho_{A,B} \), where \( \sigma_A \) is the standard deviation of the variable \( A \) and \( \rho_{A,B} \) is the coefficient of correlation between the variables \( A \) and \( B \), (A.5) can be written as:

\[ \sigma_{D \log(\text{GNI})}^2 \approx \sigma_{D \log(\text{GDP})}^2 + \left( \frac{X_t}{\text{GDP}_t} \right)^2 \cdot \sigma_{D \log(\text{PX}/\text{PM})}^2 \]

\[ + 2 \cdot \left( \frac{X_t}{\text{GDP}_t} \right) \cdot \sigma_{D \log(\text{GDP})} \cdot \sigma_{D \log(\text{PX}/\text{PM})} \cdot \rho_{D \log(\text{GDP}),D \log(\text{PX}/\text{PM})} \]  

\[ \text{(A.6)} \]
Figure A.1 below shows that the assumption that the ratio $X_t/GDP_t$ was never very different from a constant is reasonable even if a regression of this ratio on a constant and a time trend gives a significant positive coefficient on the time trend.

**Figure A.1**

![Share of export in output (X/Y)](image)
References


