CENTRAL BANK OF ICELAND

2022|88

WORKING PAPER

ANALYSING INFLATION DYNAMICS IN ICELAND USING A BAYESIAN STRUCTURAL VECTOR AUTOREGRESSION MODEL

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Also available on the Central Bank of Iceland World Wide Web site (http://www.sedlabanki.is)

ISSN 1028-9445

Analysing inflation dynamics in Iceland using a Bayesian structural vector autoregression model

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January 18, 2022

Abstract

This paper seeks to determine what drives inflation variation in Iceland and examine the extent to which local currency pricing is present. To that end we define and estimate a Bayesian structural vector autoregression model. For identification we employ the method developed by Baumeister and Hamilton (2015), defining priors on the impact matrix and on the long run behaviour of the model. We find that supply shocks and exchange rate shocks are the largest contributors in short run dynamics of inflation while foreign shocks dominate the medium and long run horizons. Our results strongly suggest that local currency pricing is largely absent. A test of robustness suggests that our results w.r.t. foreign influences on domestic inflation hold. Whether foreign demand or foreign inflation plays a larger role in determining long horizon variation in inflation seems to vary considerably over the period considered.

Keywords: Bayesian, Structural vector autoregression, Sign restrictions, Long run restrictions, inflation, international trade, small open economy.

JEL Classification Numbers: C11, C32, E31, F41

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1 Introduction

Vector autoregression (VAR) models have become ubiquitous in macroeconomic modelling, beginning with the seminal paper Macroeconomics and Reality (Sims, 1980). These atheoretical[1] models were a response to the then popular multivariate simultaneous equations models with ad hoc exogeneities. However, there are two common critiques levied at VAR models which we shall set about remedying. Firstly, there is the issue of overfitting. Overfitting translates to poor forcasting capabilities due to the estimation process assigning meaning to uninterpretable movement of the variables, relative to the model structure. This phenomenon is a result of the inherent rapid increase in parameters in a reduced VAR model as the number of variables grows. The number of parameters necessary to describe an arbitrary n-variable VAR model increases proportionally to n^2 and linearly with respect to the number of lags, p. If the time series is not sufficiently long, overfitting follows. The second problem of VAR models stems from their inherent atheoretical structure. As such and without modification it is impossible to assign economic meaning to the model's residuals. To overcome the first of these issues we can use Bayesian methods.[2] Designating appropriate priors on the parameters can reduce the tendency to overfit, thus improving, in theory, forecasting performance. If one considers forecasting performance a good metric of model quality, one expects that good priors will lead measures such as impulse functions and innovation accounting to follow suit and become a more accurate description of reality. We can attack the second problem by allowing contemporaneous effects, transforming the VAR model into a structural vector autoregression (SVAR) model. Naturally, Bayesian SVAR (BSVAR) models are not faultless. The issue of identification in SVAR models is well known and implies that the data is never informative enough on its own to allow us to infer the true parametrization of an unrestricted SVAR model. Many methods exist to mediate such concerns, all of which rest on using prior, or external, information to restrict the contemporaneous effects.[3] Unfortunately, there is no consensus on the choice of priors. Baumeister and Hamilton (2015) formalized the use of prior information to identify BSVAR models. Namely, by constructing priors directly on the impulse functions of the model in conjunction with Theil's mixed estimation (Theil, 1971) to take long run behaviour into account.

[1]These models are atheoretical in the sense that there are no assumptions on the structure of the equations themselves. However, one might consider the choice of variables, the choice of lag length employed, and the assumptions on the statistical properties of the errors as theoretical assumptions.

[2]More generally, specifying an appropriate loss function on the set parmeters reduces overfitting.

[3]See Stock and Watson (2017) for a discussion on these methods.

We augment the aforementioned method of Baumeister and Hamilton to accommodate small open economy peculiarities and estimate a 9 variable BSVAR model for Iceland. This allows us to analyse the determinants of inflation. Inflation in Iceland has been tumultous, characterized by volatility and high levels in comparison to Iceland's main trading partners, from the 1950s to the early 2010s, albeit intermittently low and stable. Furthermore, devaluation of the currency has been relatively frequent over the period.[4] As a result, researchers have put much focus on understanding inflation in Iceland. In particular, efforts have been directed towards sheding light on the effects of exchange rate shocks and foreign shocks on inflation.[5] Broadly, our aim is to elucidate which forces primarily influence inflation dynamics in Iceland. More specifically, we want to examine to what extent these dynamics differ from the findings of Thorarinsson (2020) on inflation dynamics. To that end we compare impulse responses of inflation as well as examining the forecast error variance decomposition of inflation and its historical shock decomposition. Lastly, we estimate the contemporaneous cost-push effect on import price, and examine how foreign price affects domestic inflation taking into account import price inflation. This allows us to evaluate the extent to which local currency pricing of foreign goods is present in Iceland.

The paper is structured as follows: Section 2 describes the model and data employed in the estimation. Section 3 delineates priors for all the model's parameters as well as priors on impulse response functions and restrictions the long run behaviour of the model. Section 4 details the estimation strategy and implementation. In section 5 we present our results. Finally, we briefly explore the robustness of our results in section 6.

2 Model and data

Let z_t be a $(n \times 1)$ vector of observables of interest. We posit that these variables evolve according to a dynamic structural model:

$$Az_t = Bx_{t-1} + u_t \tag{1}$$

where x_{t-1} is a $(k \times 1)$ vector of observables and a constant,[6] A is a $(n \times n)$ matrix, $B = [B_1, \ldots, B_p, B_0]$ is a $(n \times k)$ matrix, and u_t is a $(n \times 1)$ vector of

[4]See Pétursson (2018) for a brief history of inflation in Iceland; Gudmundsson et al. (2000) for details on currency devaluation in Iceland; Andersen and Gudmundsson (1998) and Snævarr (1993) (in Icelandic) for a more detailed description.

[5]See e.g. Gudmundsson (1990) (in Icelandic), Gudmundsson (2002) (in Icelandic), Pétursson (1998), Andersen and Gudmundsson (1998), and Edwards and Cabezas (2021).

[6]The constant possibly set to be identically zero.

identically and independently normally distributed stochastic elements with mean zero and variance d_{ii}. We can collect the d_{ii}'s into the diagonal of a matrix D and write $u_t \sim \mathcal{N}(0, D)$. If x_{t-1} consists of p lagged values of z_t and a constant, i.e. $x_{t-1} = [z'_{t-1}, \ldots, z'_{t-p}, 1]'$, further implying that $k = np + 1$, then equation (1) is referred to as a structural vector autoregression (SVAR). We can then write

$$A(L)z_t = u_t \tag{2}$$

or, if A is invertible:

$$A_H(L)z_t = Hu_t$$

Where $A(L) = A - \sum_{i=1}^{p} B_i L^i$, $A_H(L) = I_n - \sum_{i=1}^{n} HB_i L^i$, and $H = A^{-1}$. Thus in the case where the matrix A is invertible, we have the corresponding reduced VAR of order p

$$z_t = \Phi x_{t-1} + \varepsilon_t \tag{3}$$

where $\Phi = HB$, $\varepsilon_t = Hu_t$, and $\mathbb{E}[\varepsilon_t \varepsilon'_t] = \Omega = HDH'$. We can derive a structural vector autoregressive moving average (SVMA) representation in the case where the SVAR in equation (2) is stable:[7]

$$z_t = \Theta(L)u_t \tag{4}$$

where $\Theta(L) = \sum_{h=0}^{\infty} \Theta_h L^h$. The matrix A contains contemporaneous relationships between elements of z_t, while B relays information intertemporally. Furthermore, elements of u_t are interpreted as structural shocks, the meaning of which depends on the application.

To accommodate idiosyncrasies of the underlying data generating process in general, we can impose linear constraints on the parameters of the form

$$\beta = Rc \tag{5}$$

where $\beta = vec(B)$ or $\beta = vec(B')$. In particular, for a small open economy, like Iceland, we can demand that

$$\mathbf{b}_i = R_i c_i \tag{6}$$

where b_i is the transpose of the i'th row of the matrix B. More specifically, we insist that parameters relating effects of the domestic economy to the foreign one are identically zero. We can define $R = \oplus_{i=1}^{n} R_i$, $c = (c'_1, \ldots, c'_n)'$, with $\beta = \text{vec}(B')$ and recover the general form given in equation (5). Let us define Z, X and U to

[7] A VAR(p) model is called stable if the eigenvalues of F in the equivalent VAR(1) model $z_t = Fz_{t-1} + \varepsilon_t$ all lie inside the unit circle. We call a SVAR(p) stable if there exists a stable reduced VAR representation.

be matrices of stacked vectors of corresponding variables with dimensions $(n \times T)$, $(k \times T)$, and $(n \times T)$, respectively. We can rewrite the model given in equation (1) as

$$(I_T \otimes A)\boldsymbol{z} = (X' \otimes I_n)\, K_{nT}\beta + \boldsymbol{u}$$

where $\boldsymbol{z} = \text{vec}(Z)$, $\boldsymbol{u} = \text{vec}(U)$, $\mathbf{K}_{nT}$ is the commutation matrix,[8] and $\otimes$ is the kronecker product.[9] Inserting the linear constraint we get

$$(I_T \otimes A)\boldsymbol{z} = (X' \otimes I_n)\, R_K c + \boldsymbol{u} \tag{7}$$

where $R_K = K_{nT}R$.[10] In our application we specify a $n = 9$ variable SVAR of order $p = 2$. The structural equations stem from the small open economy IS-curve, the New Keynesian Phillips curve, the uncovered interest rate parity, a Taylor rule and Gordon's triangle model. With these specifications in mind we require measures on domestic output gap (y), domestic inflation (π), domestic interest rates (r), the real exchange rate (s), and import price inflation (π_m). The foreign economy is described with trade weighted foreign output gap, inflation and interest rates (y^*, π^*, r^*). Since our focus is on domestic inflation dynamics we augment the data set with 5 year break-even inflation expectations (π_E). A summary of our dataset can be found in table 1. The data is collected from Statistics Iceland and is at quarterly frequency. The output gap is calculated using the potential described in Daníelsson et al. (2019). Inflation is represented by per cent logarithmic difference of consumer price index. Interest rates are given by the Central bank of Iceland's (CBI) monetary policy rate measured in annual yields. Import price inflation is the per cent logarithmic difference of the import price deflator. The 5-year break even inflation is calculated from the risk adjusted ratio of interest rates on 5-year bonds and 5-year indexed bonds. Foreign variables are a trade weighted aggregation of foreign output gap,[11] inflation and interest rates. The real exchange rate, s_t, is calculated from the CBI's nominal exchange rate, the trade weighted foreign price and the CPI index. The data are all at quarterly frequency over the period 1993:Q1 – 2019:Q4.

3 Priors

Let ϕ be the vector of parameters from A, B, and D, we want to estimate. We assume that we can naturally decompose the prior density function (pdf) in the

[8] Defined such that for a $n_1 \times n_2$ matrix A we have $\text{vec}(A') = K_{n_1 n_2}\text{vec}(A)$. It has the property that for every $n_1 \times n_2$ matrix A and $m_1 \times m_2$ matrix B, we have $\mathbf{K}_{n_1 m_1}(A \otimes B)\,\mathbf{K}_{n_2 m_2} = (B \otimes A)$. Moreover, commutation matrices are orthonormal.

[9] See Lütkepohl (2005) for details.

[10] Further details can be found in the Mathematical appendix.

[11] The potential of foreign trade weighted output is found via the Hodrick-Prescott filter.

following way

$$p(\phi) = p(A, c, D) = p(A)p(D|A)p(c|A, D)$$

where we have used the restriction $\text{vec}(B') = Rc$. It follows that we want to first specify the pdf for A, then D as a possible function of A and lastly we decide upon the prior for c, which can be a function of the elements of A and D. Following Baumeister and Hamilton (2015) we use the Student's t-distribution as a default prior distribution. The t-distribution has the attractive property of being symmetrical and having fatter tails than a comparable normal distribution. We denote the Student's t-distribution with mean μ, f degrees of freedom and a scaling parameter ξ as $t_{f,\xi}(\mu)$. Furthermore, we denote a positively (negatively) truncated t-distribution as $t^{+}_{f,\xi}(\mu)$ $(t^{-}_{f,\xi}(\mu))$.[12]

3.1 Priors for contemporaneous effects

Since Iceland is a small open economy, the domestic economy does not affect the foreign economy, while the foreign economy can have a sizeable impact on the domestic economy. We achieve this by setting the relevant coefficients as identically zero. We take inspiration from Baumeister and Hamilton (2018) in using economic theory for each variable of interest to determine the priors for the contemporaneous effects other variables have on it. In setting our priors, the main references, in addition to Baumeister and Hamilton (2018), will be Thorarinsson (2020), and Galí and Monacelli (2005). Omitted priors for elements of A are to be assumed to have point mass at 0.

3.1.1 Output

Our specification of the contemporaneous dependence of output is intimetaly related to the dynamic New Keynesian IS-curve, and as such serves as aggregate demand. Both Galí and Monacelli (2005), and Thorarinsson (2020) derive an expression for the output gap, at time t, congruent to

$$y_t = \alpha_y y_{t+1|t} - \omega_y(r_t - \pi_{t+1|t}) + \theta'_y y^*_t + \mathbf{b}'_y x_{t-1} + \varepsilon_{y,t} \tag{8}$$

where $\mathbf{b}'_y x_{t-1}$ is a linear combination of lagged terms and a constant. As in Baumeister and Hamilton (2018), who note that their approach is more empirical than theoretical in nature, and whose priors are aligned with that of Doan et al. (1984) and Sims and Zha (1999), who, in turn, contend that the most recent

[12]For a given mean μ and scale parmeter ξ, we have that $t_{f,\xi}(\mu) \to \mathcal{N}(\mu, \xi^2)$ when $f \to +\infty$.

observation can be presupposed to forecast the subsequent value reasonably well, we assume that

$$y_{t+1|t} = c_y + \kappa_y y_t, \quad \pi_{t+1|t} = c_\pi + \kappa_\pi \pi_t, \quad y^*_{t+1|t} = c_{y^*} + \kappa_{y^*} y^*_t$$

where c_i is any constant, for $i \in \{y, \pi, y^*\}$, and set $\kappa = \kappa_x = 0.75$, for $x \in \{y, \pi, y^*\}$. Equation (8) becomes

$$y_t = \beta_y \pi_t - \gamma_y r_t + \theta_y y^*_t + \mathbf{b}_y x_{t-1} + \varepsilon_{y,t} \tag{9}$$

where $\gamma_y = \frac{\omega_y}{1-\alpha_y \kappa}$, $\beta_y = \frac{\omega_y \kappa}{1-\alpha_y \kappa}$, and $\theta_y = \frac{\theta'_y}{1-\alpha_y \kappa}$. Purely forward looking New Keynesian models proclaim that $\alpha_y = 1$. Including habit persistence, we get a more conservative estimate of $\alpha_y = 0.6$. The prior for an analogue of ω_y from Thorarinsson (2020), implies a mode of roughly 0.05, which gives a mode of roughly 0.1 for γ_y and 0.07 for β_y. Moreover, taking note of the different calibrations for these coefficients in the literature, as noted by Baumeister and Hamilton (2018), who calibrate their values significantly higher than Thorarinsson (2020), we decide upon the priors given in table 2. Finally, openness, a calibrated value in Thorarinsson (2020) dictating the influence of the world economy on the domestic economy, is set to 0.42, which suggests a mode of 0.57 for θ_y. However, in the benchmark specification of Galí and Monacelli (2005), foreign output gap has no effect on the domestic output gap.

3.1.2 Inflation

To determine the structure and priors of contemporaneous effects on inflation we look to the New Keynesian Phillips curve as a starting point, thus assuming the role of the aggregate supply curve. It's given by

$$\pi_t = \beta_\pi \pi_{t+1|t} + \omega_\pi mc_t$$

where mc_t is the marginal cost at time t. Both Thorarinsson (2020), and Galí and Monacelli (2005), indicate that in an open economy we can approximate marginal cost with a linear combination of the domestic output gap, the foreign output gap and the real exchange rate. We will, however, modify that representation by insisting that import prices have a contemporaneous effect and that there is no foreign contemporaneous effect that doesn't act through import prices. This is in the spirit of Matheson (2008) who argues that an appropriate model of the open-economy New Keynesian Phillips curve is one which augments the corresponding closed-economy curve with a variable capturing the tradable sector's international competitiveness, for which we employ import prices as a proxy. Batini et al. (2005) note that import price inflation, conditional on the substitutability of imports, is an indicator of external competitiveness. They stress further that import price inflation exerts pressures on inflation through the real marginal cost insofar as production inputs are imported. Moreover, this addition is motivated by the significance of import prices in the forecast error variance of inflation in Thorarinsson (2020). Finally, we augment the Phillips curve in an ad-hoc fashion by presupposing a contemporaneous effect of five year break-even inflation expectations, π_t^E. This addition is in the spirit of Pétursson (2018) and is supported by multiple estimates of the expectation augmented Phillips curve in the literature, emphasizing the importance of expectations in price formation.[13] The inflation can thus be represented as

$$\pi_t = \beta_\pi \pi_{t+1|t} + \alpha_\pi' y_t + \zeta_\pi' \pi_{F,t} + \eta_\pi' \pi_t^E + \mathbf{b}_\pi' x_{t-1} + \varepsilon_{\pi,t} \tag{10}$$

Employing our approximation for future values as we did before, we get

$$\pi_t = \alpha_\pi y_t + \zeta_\pi \pi_{F,t} + \eta_\pi \pi_t^E + \mathbf{b}_\pi x_{t-1} + \varepsilon_{\pi,t} \tag{11}$$

where reassigned coefficents have been multiplied by $(1-\beta_\pi \kappa)^{-1}$. In the canonical Phillips curve, β_π represents the discount factor, which in Thorarinsson (2020) is assumed to be 0.995. Following the calibration of Galí and Monacelli (2005), we center the priors of ζ_π, η_π, at zero. Baumeister and Hamilton (2018) cite Lubik and Schorfheide (2004) in centering their prior for α_π as 0.5. Leveraging information from Thorarinsson (2020), and Galí and Monacelli (2005), we settle for a prior with mode 0.3. The prior distributions for the Phillips curve are given in table 2. Note that we can write equation (11) as

$$y_t = g_\pi(\pi_t, \pi_{F,t}, \pi_t^E, y_t^*, x_{t-1}) + \varepsilon_{\pi,t}^*$$

where g_π is a functional and $\varepsilon_{\pi,t}^* = -\varepsilon_{\pi,t}$. It stands to reason that we should interpret a positive deviation in $\varepsilon_{\pi,t}^*$ as a positive supply shock, implying that a positive deviation in $\varepsilon_{\pi,t}$ is a negative supply shock.

3.1.3 Interest rate

For the determinants of contemporaneous monetary policy response we assume a Taylor type rule of the form

$$r_t = (1-\rho)\left(\alpha_r y_t + \beta_r \pi_t\right) + \mathbf{b}_r x_{t-1} + \epsilon_{r,t} \tag{12}$$

where $\mathbf{b}_r x_{t-1}$ is, as the reader should expect by now, a linear combination of lagged values of all relevant variables and a constant. We adopt the specification of Taylor

[13] The break-even inflation is, however, only a proxy for households' inflation expectations. See Pétursson (2018) for a discussion on the applicability of such a proxy.

(1993), assuming α_r and β_r are a priori approximately 0.5 and 1.5, respectively. For ρ, we follow Thorarinsson (2020) and Daníelsson et al. (2019) in presuming the prior is a beta distribution with mode 0.6 and standard deviation 0.0961. We include ρ as a parameter, rather than setting priors on the parameters $(1-\rho)\alpha_r$ and $(1-\rho)\beta_r$ directly, since this imposes more constraints on the model. This, however, is only the case if we adjust the priors for the lagged variables accordingly, i.e. the prior on $\mathbf{b}$ is given by $[0,\ldots,0,\rho,0,\ldots,0]$ where ρ is the parameter in front of r_{t-1}.

3.1.4 Exchange rates

Let us define the real exchange rate, s_t, as $\log\left(\xi_t \frac{P_t^*}{P_t}\right)$, where ξ_t is the trade weighted nominal exchange rate giving the home-currency price of one unit of foreign currency, P_t^* is the trade weighted foreign price level, and P_t is the CPI price level. The use of the uncovered interest rate parity, in it's original form or an augmented one, to model exchange rates has become omnipresent in New Keynesian models. One version is derived in Thorarinsson (2020) to have the form

$$s_t = \delta_S s_{t+1|t} + (1-\delta_S)s_{t-1} - \frac{1}{4}\left[(\widehat{r}_t - \widehat{\pi}_{t+1|t}) - (\widehat{r}_t^* - (\widehat{\pi}_{t+1|t}^*)\right] + \mathbf{b}_s' x_{t-1} + \varepsilon_{s,t}$$

where we define $\widehat{r}_t = r_t - \overline{r}_t$, $\widehat{r}_t^* = r_t^* - \overline{r}_t^*$, $\widehat{\pi}_t = \pi_t - \overline{\pi}$ and $\widehat{\pi}_t^* = \pi_t^* - \overline{\pi}^*$, and where $\overline{r}_t$ and $\overline{r}_t^*$ are time-varying natural rate of interests, and $\overline{\pi}$ and $\overline{\pi}^*$ are inflation targets. Using a similar argument as before we assume that a reasonable approximation is

$$\pi_{t+1}^* = c_{\pi^*} + \kappa_{\pi^*}\pi_t^*, \quad s_{t+1} = c_s + \kappa_s s_t$$

with $\kappa_s = \kappa_{\pi^*} = \kappa = 0.75$. Clearly we can decompose $\varepsilon_{s,t}$ into components consisting of the mean zero time-varying natural interest rate differential and an independent error disturbance, that is

$$\varepsilon_{s,t} = \overline{r}_{\mathcal{D},t} + \epsilon_{s,t}$$

where $\overline{r}_{\mathcal{D},t} = (\overline{r}_t^* - \overline{r}_t) - (\overline{r}^* - \overline{r})$, and $\epsilon_{s,t}$ captures the risk premium and other stochastic factors. We interpret a positive change in $\epsilon_{s,t}$ as an increase in the risk premium on Icelandic assets, and a positive change in $r_{\mathcal{D},t}$ as an equilibrium interest rate shock. Thus, the interpretation of a structural shock related directly to the real exchange rate includes a risk premium shock and natural interest rate differential shock. We have no mechanism to differentiate between these shocks when observing $\varepsilon_{s,t}$, which we subsequently refer to as an exchange rate shock. We can write

$$s_t = -\gamma_s r_t + \beta_s \pi_t + \nu_s r_t^* - \lambda_s \pi_t^* + \mathbf{b}_s x_{t-1} + a_t \tag{13}$$

where, a priori, we have the relationships $\gamma_s = \beta_s \kappa^{-1} = \nu_s = \lambda_s \kappa^{-1} = 0.25(1 - \delta_S \kappa)^{-1}$, which we collectively refer to as ω_s. From Thorarinsson (2020), we gather that δ_S is centered around 0.6, implying that $\omega_s \approx 0.5$. If, as in Galí and Monacelli (2005), $\delta_S = 1$, we get $\omega_s = 1$. We set the mode of our prior for ω_s as 0.75. The full prior distributions for γ_s, β_s, ν_s and λ_s are given in table 2.

3.1.5 Import price inflation

Our choice of contemporaneous effects on import prices reflects whether or not we assume local currency pricing. Keeping in line with our more empirical approach we assume the possibility of local currency pricing while also allowing foreign cost push effects. An advantage of this approach is that it allows us to determine the extent to which local currency pricing, which is assumed in Thorarinsson (2020), is present in Iceland. Being a hybrid model, we can use the conceptual paradigm of Gordon's triangle model (Gordon, 1988), using the domestic output gap to represent aggregate demand pressures, and foreign price and real exchange rate as supply side effects. Whence we get

$$\pi_{F,t} = \alpha_F y_t + \delta_F s_t + \lambda_F \pi_t^* + \mathbf{b}_{\pi_f} x_{t-1} + \varepsilon_{\pi_F} \tag{14}$$

It's clear that a priori we expect the sign of the parameters impacting contemporaneous explanatory variables' effects to be positive. Assuming that demand effects on import inflation are in line with demand effects on CPI inflation, we center α_F around 0.3. Viewing π^* and s as supply, or cost-push, effects we follow the specification in Thorarinsson (2020), and set modes of δ_F and λ_F to 0.5. Signaling our lack of confidence in these prior modes we allow relatively large second moments. The prior distributions can be seen in table 2.

3.1.6 The complete specification of contemporaneous effects

Since we are not interested in structural aspects of the foreign economy, and conforming to the principle of parsimony, we assume that the foreign economy is sufficiently represented, for our purposes, by a reduced VAR. We thus get the following matrix of contemporaneous effects

$$A = \begin{bmatrix} 1 & -\beta_y & \gamma_y & 0 & 0 & 0 & -\theta_y & 0 & 0 \\ -\alpha_\pi & 1 & 0 & 0 & -\zeta_\pi & -\eta_\pi & 0 & 0 & 0 \\ -(1-\rho)\alpha_r & -(1-\rho)\beta_r & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\beta_s & \gamma_s & 1 & 0 & 0 & 0 & \lambda_s & -\nu_s \\ -\alpha_F & 0 & 0 & -\delta_F & 1 & 0 & 0 & -\lambda_F & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

recalling that

$$Az_t = Bx_{t-1} + \varepsilon_t$$

and that $z_t = [y_t, \pi_t, r_t, s_t, \pi_{F,t}, \pi_{E,t}, y_t^*, \pi_t^*, r_t^*]'$, $x_{t-1} = [z_{t-1}', \ldots, z_{t-p}', 1]'$. Since we employed a truncated distribution on selected parameters, we can, in what follows, assume that all but ζ_π and η_π are positive. The joint probability density of A can be deduced from the individual specifications above as a multiple of the individual densities. Furthermore, introducing sign restrictions, at horizon 0, in the Bayesian context is equivalent to defining a prior probability density on the coefficients of $H = A^{-1}$. We can write $H = \overline{H}\det(A)^{-1} = A^{-1}$, from which it is clear that $\det(A)$ changing signs, ceterus paribus, implies that there is a plausible set of parameters which both contains elements inducing positively infinite effects of structural shocks as well as negatively infinite. A most unfortunate result, not to mention an improbable description of the economy. We thus want to restrict the determinant to a sign.[14] To determine whether $\det(A)$ is more reasonably restricted to the negative or the positive we inspect the elements of H and use economic theory to deduce the appropriate sign. The 0 horizon inflation impulse in response to a negative supply shock is given by

$$h_{2,2} = \frac{(1-\rho)\alpha_r\gamma_y}{\det(A)}$$

By reasoned assumptions we have $(1-\rho), \alpha_r, \gamma_y > 0$. In addition, theory suggests that inflation increases as a response to such a shock. As a result, we require

[14] The determinant of A is given by

$$\begin{aligned}\det(A) = {} & \gamma_y\alpha_F(1-\rho)\beta_r\zeta_\pi - \gamma_y\beta_s\delta_F(1-\rho)\alpha_r\zeta_\pi + \gamma_y\alpha_\pi(1-\rho)\beta_r \\ & + \gamma_y(1-\rho)\alpha_r + \beta_y\delta_F\gamma_s(1-\rho)\alpha_r\zeta_\pi - \beta_y\alpha_F\zeta_\pi - \beta_y\alpha_\pi \\ & + \delta_f\gamma_s(1-\rho)\beta_r\zeta_\pi - \beta_s\delta_F\zeta_\pi + 1\end{aligned}$$

$\det(A) > 0$. To achieve a desired sign restriction on a priori ambiguously signed impulse, we thus need only restrict $\overline{h}_{i,j}$.[15] We choose to restrict, $\overline{h}_{1,2} < 0$, interpreted as a negative output impulse as a response to a negative supply shock. We would like $\overline{h}_{4,3}$ to be negative to secure a rational reaction to interest rates with respect to international trade. Moreover, we want $\overline{h}_{1,7}$ to be positive, implying that, contemporaneously, an unexpected increase in foreign demand affects domestic exports positively, and consequently domestic output. Following the results from Thorarinsson (2020), we posit that $\overline{h}_{1,3}$ is negative, i.e. that an interest rate shock depresses output on impact. The effect of a monetary policy shock on interest rates is ambiguous as is. To ensure that such a shock results in higher interest rates we require $\overline{h}_{3,3}$ to be positive. We force import prices to react non-negatively to foreign markup shocks by requiring $\overline{h}_{5,8}$ to be non-negative. Lastly, we require an import price inflation shock to be positively correlated to domestic inflation, i.e. $\overline{h}_{2,5} > 0$. Note that $\overline{h}_{2,5} = (\gamma_y(1-\rho)\alpha_r + 1)\zeta_\pi$, and thus if we assume $\overline{h}_{2,5} > 0$, then that forces ζ_π to be positive, via our previous assumptions on γ_y, ρ, and α_r. These requirements imply following sign restriction matrix[16]

$$\text{SGN}(H) = \left(\begin{array}{c|ccccccccc}
 & \varepsilon_y & \varepsilon_\pi & \varepsilon_r & \varepsilon_s & \varepsilon_F & \varepsilon_E & \varepsilon_{y^*} & \varepsilon_{\pi^*} & \varepsilon_{r^*} \\
\hline
y_t & ? & - & - & ? & ? & ? & + & ? & ? \\
\pi_t & ? & + & - & + & + & ? & ? & ? & + \\
r_t & ? & + & + & + & + & ? & ? & ? & + \\
s_t & ? & ? & - & ? & ? & ? & ? & ? & ? \\
\pi_{m,t} & ? & ? & ? & ? & ? & ? & ? & + & ? \\
\pi_{E,t} & 0 & 0 & 0 & 0 & 0 & + & 0 & 0 & 0 \\
y_t^* & 0 & 0 & 0 & 0 & 0 & 0 & + & 0 & 0 \\
\pi_t^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & + & 0 \\
r_t^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +
\end{array}\right)$$

To achieve these restrictions we mimic Baumeister and Hamilton (2018) in defining a new family of distributions which they refer to as asymmetric t-distributions. For a standard Student's t distribution, $\tilde{t}_f = t_{f,1}(0)$, with f degrees of freedom, we define a distribution, $\Upsilon_{f,\xi,\chi}(\mu)\,(h)$, the probability density of which, evaluated at h, is given by:

$$p_\Upsilon(h) = k\xi^{-1}\tilde{t}_f\left(\frac{h-\mu}{f}\right)\Phi\left(\frac{\chi h}{\xi}\right)$$

[15] We abuse the meaning of a restriction slightly since our restrictions can be soft, i.e. that which is supposedly restricted can have a non-zero probability in this setup.

[16] Note that our explicit restrictions are implicit restrictions on the following entries: $\overline{h}_{3,9}$,$\overline{h}_{3,5}$,$\overline{h}_{3,4}$,$\overline{h}_{2,9}$,$\overline{h}_{2,5}$,$\overline{h}_{2,4}$,$\overline{h}_{2,3}$.

where Φ is cumulative distribution function of the standard normal distribution, ξ is a scaling parameter, μ is a location parameter, χ is shape parameter, and k is defined such that $\Upsilon_{f,\xi,z}(\mu)(\cdot)$ has unit measure. Moreover, for each prior we specify a weight $w_{i,j}$, which determines each priors weight in the target function q_H, which determines the acceptance probability in our Metropolis-Hasting Markov chain Monte Carlo algorithm and is defined in section 4. We set $w_{i,j} = 100$ in all cases. A prior on $\overline{h}_{i,j}$ is equivalent to a joint prior on a subset of elements from A. More specifically, it is the joint prior on the following expressions:

$$
\begin{aligned}
\overline{h}_{1,2} &= \gamma_y(1-\rho)\beta_r - b_y \\
\overline{h}_{1,3} &= \zeta_\pi\left(\beta_s\delta_F\gamma_y + \beta_y\delta_F\gamma_s\right) - \gamma_y \\
\overline{h}_{1,7} &= \theta_y\left[\zeta_\pi\delta_F\left(\gamma_s(1-\rho)\beta_r - \beta_s\right) + 1\right] \\
\overline{h}_{2,5} &= \zeta_\pi\left(\gamma_y(1-\rho)\alpha_r + 1\right) \\
\overline{h}_{3,3} &= \zeta_\pi\left(\beta_s\delta_F + \alpha_F\beta_y\right) + \alpha_\pi\beta_y - 1 \\
\overline{h}_{4,3} &= \zeta_\pi\alpha_F\left(\beta_s\gamma_y - \beta_y\gamma_s\right) + \alpha_\pi\beta_s\gamma_y + \gamma_s\left(1-\alpha_\pi\beta_y\right) \\
\overline{h}_{5,8} &= -\left[(1-\rho)\gamma_y\left(\delta_F\lambda_s - \lambda_F\right)\left(\alpha_\pi\beta_r + \alpha_r\right)\right] \\
&\quad - \left[\left(1-\alpha_\pi\beta_y\right)\delta_F\lambda_s + \left(\alpha_\pi\beta_y - 1\right)\lambda_F\right]
\end{aligned}
$$

Table 3 gives the parametrization of prior densities for $\overline{H}$. To determine the exact parameters of the densities we sample vectors from the joint prior density of A, examine the moments of $\overline{h}_{i,j}$ and set f,ξ and z such that we minimally deviate from these moments while also introducing asymmetry in the distribution to increase the likelihood of getting the designated sign on the impulse responses at horizon $h = 0$.

The full model is given by

$$
\begin{aligned}
y_t &= \beta_y\pi_t - \gamma_y r_t + \theta_y y_t^* + \mathbf{b}_y x_{t-1} + \varepsilon_{y,t} &(15)\\
\pi_t &= \alpha_\pi y_t + \zeta_\pi\pi_{F,t} + \eta_\pi\pi_t^E + \mathbf{b}_\pi x_{t-1} + \varepsilon_{\pi,t} &(16)\\
r_t &= (1-\rho)\alpha_r y_t + (1-\rho)\beta_r\pi_t + \mathbf{b}_r x_{t-1} + \epsilon_{r,t} &(17)\\
s_t &= \beta_s\pi_t - \gamma_s r_t - \lambda_s\pi_t^* + \nu_s r_t^* + \mathbf{b}_s x_{t-1} + \varepsilon_{s,t} &(18)\\
\pi_{F,t} &= \alpha_F y_y + \delta_F s_t + \lambda_F\pi_t^* + \mathbf{b}_F x_{t-1} + \varepsilon_{F,t} &(19)\\
\pi_{E,t} &= \mathbf{b}_E x_{t-1} + \varepsilon_{E,t} &(20)\\
y_t^* &= \mathbf{b}_{y^*} x_{t-1} + \varepsilon_{y^*,t} &(21)\\
\pi_t^* &= \mathbf{b}_{\pi^*,t} x_{t-1} + \varepsilon_{\pi^*,t} &(22)\\
r_t^* &= \mathbf{b}_{r^*} x_{t-1} + \varepsilon_{r^*,t} &(23)
\end{aligned}
$$

3.2 Priors for $D|A$

We mimic Baumeister and Hamilton (2015) in choosing priors for the covariance matrix of u_t, by setting the prior as a product of inverse-gamma distributions, the natural conjugate prior. We thus have $p(D|A) = \Pi_{i=1}^{n} p(d_{ii}|A)$, where

$$p(d_{ii}^{-1}) = \begin{cases} \frac{\tau_i^{\kappa_i}}{\Gamma(\kappa_i)} (d_{ii}^{-1})^{\kappa_i - 1} \exp(-\tau_i d_{ii}^{-1}) & \text{if } d_{ii}^{-1} > 0 \\ 0 & \text{otherwise} \end{cases}$$

Let $\mathfrak{S}$ be the variance matrix of univariate autoregressions for the elements of y_t, then we set the mean of the prior for d_{ii} as $\frac{1}{\mathbf{a}_i' \mathfrak{S} \mathbf{a}_i}$. The weight of the prior is set as $\kappa_i = 2$.[17]

3.3 Priors for $c|A, D$

We define the conditional prior for c by specifying a non-controversial prior for B, which we relate to c via our linear constraint. Following Baumeister and Hamilton, we define the conditional probability density function of the prior for B, given A and D, as

$$p(B|A, D) = \Pi_i p(\mathbf{b}_i|A, D) = \Pi_i \phi(\mathbf{b}_i; \mathbf{m}_i, d_{ii}, \mathbf{M}_i)$$

where $\phi(\mathbf{b}_i; \mathbf{m}_i, d_{ii}, \mathbf{M}_i)$ is the pdf of a multivariate Gaussian distribution with mean $\mathbf{m}_i$ and variance $d_{ii}\mathbf{M}_i$, evaluated at $\mathbf{b}_i$. In particular, the prior is given by

$$p(\mathbf{b}_i|A, D) = (2\pi)^{\frac{k}{2}} \det(d_{ii} M_i)^{-\frac{1}{2}} \exp\left[-0.5 \mathfrak{m}(\mathbf{b}_i, \mathbf{m}_i, d_{ii} M_i)\right] \quad (24)$$

where

$$\mathfrak{m}(x, y, F) = (x - y)' F^{-1} (x - y)$$

is the squared Mahalanobis distance, defined for an arbitrary n-dimensional vector pair x, y and an arbitrary $(n \times n)$ invertible matrix F. More succinctly, we can write

$$\beta \sim N(\mathbf{m}, \mathbf{M}) \quad (25)$$

where $\beta = vec(B')$, $\mathbf{m} = [\mathbf{m}_1', \ldots, \mathbf{m}_i', \ldots, \mathbf{m}_n']'$, and $\mathbf{M}$ is a block diagonal matrix given by the direct sum $\mathbf{M} = \oplus_{i=1}^{n} d_{ii} \mathbf{M}_i$. Recall that our base model, given by equation (1) is constrained by $\beta = Rc$, where R is given by the direct sum $R = \oplus_{i=1}^{n} R_i$, where $\mathbf{b}_i = R_i c_i$, and $c = [c_1', \ldots, c_n']'$. As illuminated by the discussion in Lütkepohl (2005) there exists a matrix S, for a given R satisfying $\beta = Rc$, such that

$$S\text{vec}(B) = c^*$$

[17] Since the prior mean of d_{ii}^{-1} is given by $\frac{\kappa_i}{\tau_i}$, this specification uniquely determines τ_i and κ_i.

In our case where linear restrictions are those of forcing selected parameters to be zero, we have $R' = S$, $c^* = c$, and $S \cdot R = I_{\dim(c)}$,[18] as the reader can check. Furthermore, we can identify $S_i = R_i'$. Thus we can write

$$c_i \sim N(S_i \mathbf{m}_i, S_i d_{ii} \mathbf{M}_i S_i') = N(\mathbf{m}_{c,i}, d_{ii} \mathbf{M}_{c,i})$$

or

$$c \sim N(S\mathbf{m}, S\mathbf{M}S') = N(\mathbf{m}_c, \mathbf{M}_c)$$

and $p(c|A, D) = \prod_{i=1}^{n} p(c_i|A, D)$. In accordance with Baumeister and Hamilton (2018) we set $\mathbf{m}_i = 0.75\mathfrak{J}_9' \mathbf{a_i}$ for all i except the one relating to the interest rate, where

$$\mathfrak{J}_{9\times 9p} = [\mathcal{I}_9 \quad \mathbf{0}_{9\times 9(p-1)+1}]$$

implying that we hesitantly anticipate that at time t, there is little useful information beyond $t-1$. For the equation governing the evolution of interest rates we set the prior such that ρ is the prior coefficient for a lagged value of the interest rate, while others are set to 0. Finally, $\mathbf{M}_i$ is the confidence we have in our priors for B. Conforming with Baumeister and Hamilton (2015), who decide $\mathbf{M}_i$ in accordance with the Minnesota priors, we calibrate the hyperparameters as $\lambda_0 = 0.1$, $\lambda_1 = 1$, $\lambda_2 = 100$.[19]

3.3.1 Pseudo Long run restrictions and SVMA representation

Consider again the model

$$Az_t = Bx_{t-1} + u_t$$

Assume that $B = [B_1, \ldots, B_p, B_0]$, $x_{t-1} = [z_{t-1}', \ldots, z_{t-p}', 1]'$, and u_t is a differentiable martingale difference sequence. We can rewrite the model as

$$Az_t = B_0 + B_1 z_{t-1} + \cdots + B_p z_{t-p} + u_t$$

Assume that A is invertible and define $\Phi(L) = I_n - \Phi_1 L - \cdots - \Phi_p L^p$, where $\Phi_i = A^{-1}B_i$ and $B_0' = A^{-1}B_0$. We then have the representation

$$\Phi(L) z_t = B_0' + \varepsilon_t$$

where $\varepsilon_t = A^{-1}u_t$. Define further $\Theta(L) = \sum_{i=0}^{\infty} \Theta_i L^i$ which satisfies

$$\Theta(L)\Phi(L) = I_n$$

[18] We denote by $\dim(c)$ the dimension of the underlying space c lives in, i.e. $c \in \mathbb{R}^{\dim(c)}$.

[19] See Baumeister and Hamilton (2018) and Lütkepohl (2005, p. 225) for details.

Assuming the existence of $\Theta(L)$, this implies a SVMA representation:

$$z_t = \mu + \Theta(L)\varepsilon_t$$

Where $\mu = \Theta(1)B_0'$. Comparing coefficients of the expression

$$\left(\Theta_0 + \Theta_1 L + \cdots + \Theta_i L^i + \dots\right)(I_n - \Phi_1 L - \cdots - \Phi_p L^p) = I_n$$

we get

$$\Theta_0 = I$$

$$\Theta_i = \sum_{j=1}^{i} \Theta_{i-j}\Phi_j$$

$$\Theta_i = \sum_{j=1}^{i} \Theta_{i-j}A^{-1}B_j$$

and $B_j = 0$ for $j > p$. Thus existence of $\Theta(L)$ is secured. The impulse response function (IRF) at horizon h of the ith element of variable z, to the jth shock, $u_{t,j}$, of unit size, is defined as

$$\frac{\delta z_{i,t+h}}{\delta u_{j,t}} = \frac{\delta z_{i,t+h}}{\delta u_t}\frac{\delta u_t}{\delta u_{t,j}} = \left(\Theta_h A^{-1}\right)_{i,j}$$

Let Y_t be such that $\Delta Y_t = z_t$ and define the long run impact of a shock u_t by

$$\lim_{h\to\infty} \frac{\delta}{\delta u_t}(Y_{t+h} - Y_t) = \lim_{h\to\infty} \sum_{i=0}^{h} \frac{\delta z_{t+i}}{\delta u_t} = \lim_{h\to\infty} \sum_{i=0}^{h} \Theta_i A^{-1} = \Theta(1)A^{-1}$$

By definition $\Theta(1) = \Phi(1)^{-1}$ and we get

$$\lim_{h\to\infty} \frac{\delta}{\delta u_t}(Y_{t+h} - Y_t) = (I_n - \Phi_1 - \cdots - \Phi_p)^{-1}A^{-1} = (A - B_1 - \cdots - B_p)^{-1}$$

Where we used that $(X_1X_2)^{-1} = X_2^{-1}X_1^{-1}$ and $\Phi_i = A^{-1}B_i$, and assumed appropriate invertibility. Long run restrictions are thus restrictions on the matrix $Q = (A - B_1 - \cdots - B_p)^{-1}$. Assuming that no shock has a non-zero cumulative effect on GDP, other than a supply shock, we get restrictions on the elements of $\Phi(1)$ such that[20,21]

$$0 = q_i^{\mathcal{J}} = \sum_{j=1}^{p} b_{2,i,j}, \quad i \in \mathcal{R} \tag{26}$$

[20] This follows from the facts that the inverse can be calculated from elementary matrix operations and that a row with a single element will remain a row with a single element in such a procedure. For an arbitrary matrix with the i-th row consisting of the vector $[0, \dots, 0, x_k, 0, \dots, 0]$, where x_k is the k-th element, the k-th row of the inverse will consist of a vector $[0, \dots, 0, x_i^*, 0, \dots, 0]$.

[21] An element in the k-th row, i-th column of the matrix B_j is written $b_{k,i,j}$.

where $\mathcal{R}$ is a set of restriction indices. For our purposes, we set $\mathcal{R} = \{2, \ldots, n\}$. Next, we define a prior on $\sum_{i=1}^{p} b_{2,i,j}$, for $i \in \mathcal{R}$, as

$$\sum_{j=1}^{p} b_{2,i,j} \bigg| A \sim N(q_i^{\mathcal{J}}, d_{ii}V_i)$$

where V_i is the confidence we have in each long run restriction. Using Theil's mixed estimation method, we can define, for appropriate i's, a stochastic linear restriction which can depend on A and D, by

$$r_i = R_i b_i + v_i$$

where $v_i \sim N(0, d_{ii}V_i)$, allowing us to extend our observation vector with $\sum_{i=1}^{n} h_i$ pseudo observations, where h_i is the dimension of r_i. We augment our observables by adding appropriately

$$r_i' P_{V,i} = R_i' P_{V,i} b_i$$

where P_V is the Cholesky decomposition of V_i. Following Baumeister and Hamilton (2018) we set $V_i = 0.1$. Heed, however, that in our representation output is observed as a percentage deviation from potential output. As such, it's discrete integral is $I(0)$. Long run restrictions should thus, in theory, not impact identification. In practice, however, using long run restriction imposes stricter identification which reduces variance of posteriors. The proper interpretation of the restrictions is that the cumulative effects of the aforementioned shocks on the output gap is null.

4 Methodology

We employ a slightly modified version of the Baumeister-Hamilton algorithm, adjusted for Icelandic idiosyncrasies. Namely we impose that the world economy is not affected at all by domestic shocks. Proposition 1, which is virtually identical to that of Baumeister and Hamilton (2015), implies that the BH algorithm produces the desired results.[22]

Proposition 1. *Let us assume that z_t evolves according to equation (1), and the relevant quantities have properties as previously stated. Let us denote the priors for c and d_{ii}^{-1} by $\phi(c; m_c, M_c)$ and $\gamma(d_{ii}^{-1}; \kappa_i, \tau_i)$, respectively. Then the posterior of A satisfies*

$$p(A|Y_T) \propto \frac{p(A)\det(A\hat{\Omega}A')^{\frac{T}{2}}}{\prod_{i=1}^{n}(\frac{2}{T}\tau_i^*)^{\kappa_i^*}} \prod_{i=1}^{n} \frac{\det(M_{c,i}^*)^{\frac{1}{2}}}{\det(M_{c,i})^{\frac{1}{2}}} \frac{\tau_i^{\kappa_i}}{\Gamma(\kappa_i)} \Gamma(\kappa_i^*) \tag{27}$$

[22]The proof of Proposition 1 is given in the Mathematical appendix.

where $\kappa_i^* = \kappa_i + \frac{T}{2}$, $\tau_i^* = \tau_i + \frac{\zeta_i}{2}$. *Moreover, the posterior for* c *is given by*

$$\phi(c; m_c^*, M_c^*) = \Pi_{i=1}^n \phi(c_i; m_{c,i}^*, M_{c,i}^*) \tag{28}$$

where $\tilde{m}_{c,i} = \left(\tilde{X}_i'\tilde{X}_i\right)^{-1}\left(\tilde{X}_i'\tilde{z}_i\right)$, $\tilde{M}_{c,i} = \left(\tilde{X}_i'\tilde{X}_i\right)^{-1}$, *and* $\tilde{X}_i$ *and* $\tilde{z}_i$ *are defined as*

$$\underset{(T+q_i\times 1)}{\tilde{z}_i} = \begin{bmatrix} z_1'a_i \\ \vdots \\ z_t'a_i \\ \vdots \\ z_T'a_i \\ P_{c,i}'\mathbf{m}_{c,i} \end{bmatrix}, \qquad \underset{(T+q_i\times q_i)}{\tilde{X}_i} = \begin{bmatrix} x_1'R_i \\ \vdots \\ x_t'R_i \\ \vdots \\ x_T'R_i \\ P_{c,i}' \end{bmatrix} \tag{29}$$

The posterior of the elements of D, d_{ii}^{-1}, *is given by* $\gamma(d_{ii}^{-1}; \kappa_i^*, \tau_i^*)$. *Finally, the implied target function is given by*

$$\begin{aligned} q(A) = {} & \log p(A) + \frac{T}{2}\log[\det(A\hat{\Omega}A')] \\ & - \sum_{i}^{n}\left(\kappa_i + \frac{T}{2}\right)\log\left[\left(\frac{2\tau_i}{T} + \frac{\zeta_i}{T}\right)\right] + \sum_{i=1}^{n}\kappa_i\log\tau_i \\ & + \sum_{i=1}^{n}\frac{1}{2}\left[\log\frac{|M_{c,i}^*|}{|M_{c,i}|} + \log\frac{\Gamma(\kappa_i + \frac{T}{2})}{\Gamma(\kappa_i)}\right] \end{aligned} \tag{30}$$

If $M_{c,i}^*$ *and* κ^* *don't depend on* A, *then we get*

$$\begin{aligned} q(A) = {} & \log p(A) + \frac{T}{2}\log[\det(A\hat{\Omega}A')] \\ & - \sum_{i=1}^{n}\left(\kappa_i^*\log\tau_i^* - \kappa_i\log\tau_i\right) \end{aligned} \tag{31}$$

□

Note that the priors on long run behaviour are omitted in the proposition. Since such priors, via Theil's mixed estimator, can be viewed as pseudo-observations, the proposition holds with the appropriate changes to $\tilde{z}_i$ and $\tilde{X}_i$. Furthermore, appending the priors for the impact matrix H to the target function in Proposition 1 gives the target function we employ in our algorithm, namely

$$q_H(A) = q(A) + \sum_{(i,j)\in\mathcal{R}_H} w_{i,j}\log\left(\Upsilon_{(i,j)}(H)\right) \tag{32}$$

where $\Upsilon_{(i,j)}(H)$ is a generalized prior for the impact of shock j on variable i at horizon $h = 0$, $\mathcal{R}_H$ is a set of indices and $w_{i,j}$ are weights.

5 Results

We use the BSVAR model described above, estimated via an augmented version of the Baumeister and Hamilton (2015) algorithm, to produce impulse responses, forecast error variance decompositions and historical variance decompositions. It is evident from Figures 1 and 2, depicting prior and posterior densities of parmeters influencing contemporaneous effects, that, in general, the data is informative on the posteriors. In almost all cases the variance decreases and the mode shifts. The mean and standard deviation of the posteriors can be found in Table 4. Figure 3 displays the cumulative impulse response functions of inflation to structural shocks. Of note here, contrary to results of Thorarinsson (2020), is the marked difference in the effect of foreign price inflation and import price inflation. In fact, when the transitory effects have died down, the cumulative impact of foreign price inflation shock is roughly an order of magnitude bigger than the impact of import price inflation. This implies that import price inflation contains little information for domestic prices when we adjust for foreign price inflation. Evidence of this can be seen clearly in figure 5, where the ratio of the impulse response of inflation to ε_{π^*} and ε_{π_m} is shown. In addition, from figure 4, which displays the cumulative impulse responses of import inflation, we see that the cumulative effect of foreign shocks is comparatively large. In particular the cost-push effect dominates import price formation. This is further supported by our estimate of the contemporaneous cost-push effect, λ_F, with a posterior distribution which is strictly positive, has a mean around unity, and implies that the effect of world price on import inflation is considerable. A similar story is told by the forecast error variance decomposition, where foreign effects make up 20%, 77% and 84% percent of the deviation in inflation at horizons 4, 16, and 32 quarters, respectively.[23] The average explanation power of the shocks on deviations in inflation can be seen in table 5. Over the 8 year horizon, foreign demand constitutes 49.2%, foreign price inflation 13.6%, real exchange rates 13.1%, of inflation variance, on average. Our findings strongly suggest that foreign events dominate domestic ones in price decisions and that inflation dynamics in Iceland are heavily influenced by foreign developments in general.[24] In particular, and as can be seen in figure 6, medium and long term dynamics are predominantly determined by evolution of the foreign economy while short term dynamics are dominated by domestic effects. Observe, however, that this is partially by design since strictly foreign variables are excluded from contemporaneous effects on inflation. The comparison to Thorarinsson (2020) is slightly compromised, however, by the fact that they assume local currency

[23] A more detailed breakdown w.r.t horizons can be seen in figure 6.

[24] This is in line with findings report in Box 2 of the Central Bank of Iceland's fourth issue of the Monetary bulletin in 2015.

pricing. A caveat to our findings that bears mentioning is that the global financial crisis is included in the data set, and as figures 8, 9, and 10 indicate, the model seems to deem foreign shocks to be responsible for the rise and fall of inflation during that period.[25] Therefore, presupposing that the results are representative of the current state of the Icelandic economy might overstate the impact of foreign shocks. Moreover, by inspection we deduce that world demand has been more instrumental in price formation before, and during, the global financial crisis than after. This is harmoneous with the findings of Breedon et al. (2021) who observe that information content of trading activity and sensitivity to macro news in the FX market are heavily affected by the capital controls put in effect in Iceland in the wake of the global financial crisis.

6 Robustness

To determine to what extent the inclusion of the global financial crisis is driving our results we re-estimate the same model without modification over the period 2011:Q1-2019Q4, altering the time frame of the data and excluding the global financial crisis. As can be seen in figure 7, foreign effects are still dominant in this period. However, now foreign price inflation explains 58% of forecast variance on average over 8 years, while world demand explains merely 8%. The estimate over this period is qualitatively similar but the data much less informative. This is to be expected with a smaller number of observations. As a result, caution should be taken when drawing conclusions from this comparison estimate.

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7 Mathematical appendix

7.1 Proof of proposition 1

The proof strategy is to write $p(Z_T, A, c, D)$ in two different ways. From there we can derive an expression which is proportional to $p(A|Z_T)$. Assuming that A is non-singular, the likelihood of Y_T, can be calculated from equation (3) as

$$\mathcal{L}(\cdot|Z) = (2\pi)^{-\frac{Tn}{2}} \det(\Omega)^{-\frac{T}{2}} \prod_{t=1}^{T} \exp\left[-\frac{1}{2}\mathfrak{m}(z_t, A^{-1}Bx_{t-1}, \Omega)\right] \tag{33}$$

subject to the constraint that $\mathbf{b}_i = R_i c_i$. By assumption we have $\Omega = A^{-1}D(A^{-1})'$, and it's well known that the Mahalanobis distance yields the following relationship

$$\mathfrak{m}(Ay, Ax, F) = \mathfrak{m}(y, x, A^{-1}F(A^{-1})')$$

We thus get

$$\mathcal{L}(\cdot|Z) = (2\pi)^{-\frac{Tn}{2}} \det(A^{-1}D(A^{-1})')^{-\frac{T}{2}} \prod_{t=1}^{T} \exp\left[-\frac{1}{2}\mathfrak{m}(Az_t, Bx_{t-1}, D)\right] \tag{34}$$

It is further a well known property of the Mahalanobis distance that for a diagonal matrix D, with diagonal elements d_{ii}, we can write

$$\mathfrak{m}(Ay_t, Bx_{t-1}, D) = \sum_{i=1}^{n} \frac{(a_i' z_t - b_i' x_{t-1})^2}{d_{ii}}$$

Where a_i' and b_i' are row vectors of A and B, respectively. Using $\det(A^{-1}D(A^{-1})') = \det(A)^{-2}\det(D)$ and rearranging, we get

$$\mathcal{L}(\cdot|Y) = (2\pi)^{-\frac{Tn}{2}} \det(D)^{-\frac{T}{2}} |\det(A)|^{T} \prod_{i=1}^{n} \exp\left[-\sum_{t=1}^{T} \frac{(a_i' z_t - b_i' x_{t-1})^2}{2d_{ii}}\right] \tag{35}$$

Inserting the linear constraint, $b_i = R_i c_i$, or equivalently $b_i' = c_i' R_i' = c_i' S_i$ yields

$$\mathcal{L}(\cdot|Y) = (2\pi)^{-\frac{Tn}{2}} \det(D)^{-\frac{T}{2}} |\det(A)|^{T} \prod_{i=1}^{n} \exp\left[-\sum_{t=1}^{T} \frac{(a_i' z_t - c_i' S_i x_{t-1})^2}{2d_{ii}}\right] \tag{36}$$

Recall that $P_{c,i}$ denotes the Cholesky factor of $M_{c,i}$, satisfying the property that $M_{c,i}^{-1} = P_{c,i}P_{c,i}'$. Defining $X_i^* = P_{c,i}'$ and $y^* = P_{c,i}'\mathbf{m}_{c,i}$, we can write the prior for c_i as

$$p(c_i|A, D) = (2\pi)^{-\frac{q_i}{2}} \det(d_{ii}M_{c,i})^{-\frac{1}{2}} \exp\left[-\frac{1}{d_{ii}2}\left(y_i^* - X_i^* c_i\right)'\left(y_i^* - X_i^* c_i\right)\right] \tag{37}$$

With the goal of combining the likelihood and the prior, let us now define a $(T + q_i) \times 1$ vector $\tilde{z}_i$ and $(T + q_i) \times q_i$ matrix $\tilde{X}_i$ as

$$\underset{(T+q_i\times 1)}{\tilde{z}_i^*} = \begin{bmatrix} z_1' a_i \\ \vdots \\ z_t' a_i \\ \vdots \\ z_T' a_i \\ P_{c,i}'\mathbf{m}_{c,i} \end{bmatrix}, \qquad \underset{(T+q_i\times q_i)}{\tilde{X}_i^*} = \begin{bmatrix} x_1' R_i \\ \vdots \\ x_t' R_i \\ \vdots \\ x_T' R_i \\ P_{c,i}' \end{bmatrix}$$

where $q_i \leq k$ is the dimension of c_i. Conjoining equations (36) and (37) we get

$$p(Z|A,c,D)p(c|A,D) = \aleph \cdot \det(D)^{-\frac{T}{2}} |\det(A)|^T \cdot \left(\prod_{i=1}^{n} \det(d_{ii} M_{c,i})^{-\frac{1}{2}} \exp\left[-\frac{\mathfrak{m}(\tilde{z}_i, \tilde{X}_i \mathbf{c}_i, I_{q_i})}{d_{ii} 2} \right] \right) \tag{38}$$

where $\aleph = (2\pi)^{-\frac{Tn+\sum_{i=1}^{n} q_i}{2}}$ is a constant since we assume that T, n, and q_i, for all i, are predetermined. Without loss of efficiency, the system $\tilde{Z} = \tilde{X}c$, where $\tilde{Z}$ is the collection of $\tilde{z}_i$'s and $\tilde{X}$ is the collect of $\tilde{X}_i$'s, can be estimated line by line, giving us the estimate

$$\mathbf{m}^*_{c,i} = \left(\tilde{X}_i' \tilde{X}_i \right)^{-1} (\tilde{X}_i)' \tilde{z}_i$$

Using the fact that $\tilde{z}_i - \tilde{X}_i \mathbf{m}^*_{c,i}$ is orthogonal to $\tilde{X}_i$, we can write[26]

$$\mathfrak{m}(\tilde{z}_i, \tilde{X}_i \mathbf{c}_i, I_{q_i}) = \zeta_i + (\mathbf{m}^*_{c,i} - \mathbf{c}_i)'(M^*_{c,i})^{-1}(\mathbf{m}^*_{c,i} - \mathbf{c}_i)$$

where $M^*_{c,i} = \left(\tilde{X}_i' \tilde{X}_i \right)^{-1}$ and $\zeta_i = (\tilde{z}_i - \tilde{X}_i \mathbf{m}^*_{c,i})'(\tilde{z}_i - \tilde{X}_i \mathbf{m}^*_{c,i})$. Equation (38) becomes

$$p(Y|A,c,D)p(c|A,D) = \aleph \det(D)^{-\frac{T}{2}} |\det(A)|^T \left(\prod_{i=1}^{n} \det(d_{ii} M_{c,i})^{-\frac{1}{2}} \right) \cdot \prod_{i=1}^{n} \exp\left[-\frac{\zeta_i + (\mathbf{m}^*_{c,i} - \mathbf{c}_i)'(M^*_{c,i})^{-1}(\mathbf{m}^*_{c,i} - \mathbf{c}_i)}{2 d_{ii}} \right] \tag{39}$$

Recall that the conditional prior for $D|A$ is given by

$$p(D|A) = \prod_{i=1}^{n} p(d_{ii}|A)$$

where

$$p(d_{ii}^{-1}) = \begin{cases} \frac{\tau_i^{\kappa_i}}{\Gamma(\kappa_i)} (d_{ii}^{-1})^{\kappa_i - 1} \exp(-\tau_i d_{ii}^{-1}), & \text{for } d_{ii}^{-1} \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

We denote this density with $\gamma(d_{ii}^{-1}; \kappa_i, \tau_i)$ and refer to it as the gamma density with shape κ_i and rate τ_i, evaluated at d_{ii}^{-1}. Further recall that we denote the probability density function of a multivariate normal distribution with mean μ and covariance matrix Σ, evaluated at x, with $\phi(x; \mu, \Sigma)$. Now multiplying equation

[26]See Baumeister and Hamilton (2014) for details.

(39) with the conditional prior for $D|A$ and the prior for A we get and expression for $p(Y_T, A, c, D) = p(A)P(D|A)p(Y|A, c, D)p(c|A, D)$:

$$
\begin{aligned}
p(Y_T, A, c, D) = \aleph p(A)|\det(A)|^T \prod_{i=1}^{n} \frac{\tau_i^{\kappa_i}}{\Gamma(\kappa_i)} (d_{ii}^{-1})^{\kappa_i - 1} \exp(-\tau_i d_{ii}^{-1}) \\
\cdot \prod_{i=1}^{n} d_{ii}^{-\frac{T}{2}} \prod_{i=1}^{n} d_{ii}^{-\frac{q_i}{2}} \det(M_{c,i})^{-\frac{1}{2}} \\
\cdot \prod_{i=1}^{n} \exp\left[-\frac{\zeta_i + (\mathbf{m}_{c,i}^* - \mathbf{c}_i)'(M_{c,i}^*)^{-1}(\mathbf{m}_{c,i}^* - \mathbf{c}_i)}{2d_{ii}}\right] \\
= \aleph_2 p(A)|\det(A)|^T \prod_{i=1}^{n} \frac{\tau_i^{\kappa_i}}{\Gamma(\kappa_i)} (d_{ii}^{-1})^{\kappa_i - 1} \exp(-\tau_i d_{ii}^{-1}) \\
\cdot \prod_{i=1}^{n} d_{ii}^{-\frac{T}{2}} \prod_{i=1}^{n} d_{ii}^{-\frac{q_i}{2}} \det(M_{c,i})^{-\frac{1}{2}} \\
\cdot \prod_{i=1}^{n} \exp\left(-\frac{\zeta_i}{2d_{ii}}\right) d_{ii}^{\frac{q_i}{2}} \det(M_{c,i}^*)^{\frac{1}{2}} \phi(\mathbf{c}_i; \mathbf{m}_{c,i}^*, d_{ii} M_{c,i}^*)
\end{aligned}
$$

where $\aleph_2 = \aleph \cdot (2\pi)^{\frac{\sum_{i=1}^n q_i}{2}} = (2\pi)^{-\frac{Tn}{2}}$. Define $\tau_i^* = \tau_i + \frac{\zeta_i}{2}$ and $\kappa_i^* = \kappa_i + \frac{T}{2}$. We can further rewrite the rightmost side of the last equation as

$$
\begin{aligned}
p(Y_T, A, c, D) = \aleph_2 p(A)|\det(A)|^T \prod_{i=1}^{n} \frac{\tau_i^{\kappa_i}}{\Gamma(\kappa_i)} (d_{ii}^{-1})^{\kappa_i - 1} \exp(-\tau_i d_{ii}^{-1}) \\
\cdot \prod_{i=1}^{n} d_{ii}^{-\frac{T}{2}} \prod_{i=1}^{n} \det(M_{c,i})^{-\frac{1}{2}} \\
\cdot \prod_{i=1}^{n} \exp(\tau_i d_{ii}^{-1}) \exp(-\tau_i^* d_{ii}^{-1}) \det(M_{c,i}^*)^{\frac{1}{2}} \phi(\mathbf{c}_i; \mathbf{m}_{c,i}^*, d_{ii} M_{c,i}^*)
\end{aligned}
$$

and finally

$$\begin{aligned}
p(Y_T, A, c, D) = \aleph_2 p(A)|\det(A)|^T \\
\cdot \prod_{i=1}^{n} \frac{\tau_i^{\kappa_i}}{\Gamma(\kappa_i)} (d_{ii}^{-1})^{\frac{T}{2}+\kappa_i-1} \exp(-\tau_i^* d_{ii}^{-1}) \\
\cdot \prod_{i=1}^{n} \frac{\det(M_{c,i}^*)^{\frac{1}{2}}}{\det(M_{c,i})^{\frac{1}{2}}} \phi(c_i; \mathbf{m}_{c,i}, d_{ii} M_{c,i}^*) \\
= \aleph_2 p(A)|\det(A)|^T \prod_{i=1}^{n} \frac{\det(M_{c,i}^*)^{\frac{1}{2}}}{\det(M_{c,i})^{\frac{1}{2}}} \frac{\tau_i^{\kappa_i}}{\Gamma(\kappa_i)} \frac{\Gamma(\kappa_i^*)}{(\tau_i^*)^{\kappa_i}} \qquad (40) \\
\cdot \prod_{i=1}^{n} \gamma(d_{ii}^{-1}; \kappa_i^*, \tau_i *) \phi(\mathbf{c}_i; \mathbf{m}_{c,i}^*, d_{ii} M_{c,i}^*)
\end{aligned}$$

We can alternatively decompose $p(Y_T, A, c, D)$ as

$$p(Y_T, A, c, D) = p(Y_T)p(A|Y_T)p(D|A, Y_T)p(c|A, D, Y_T)$$

By inspection it is clear that we can associate $\prod_{i=1}^{n} \phi(c_i; \mathbf{m}_{c,i}, d_{ii} M_{c,i}^*)$ with $p(c|A, D, Y_T)$, and $\prod_{i=1}^{n} \gamma(d_{ii}^{-1}; \kappa_i^*, \tau_i^*)$ with $p(D|A, Y_T)$. Whence we deduce that

$$p(Y_T)p(A|Y_T) \propto \aleph_2 p(A)|\det(A)|^T \prod_{i=1}^{n} \frac{\det(M_{c,i}^*)^{\frac{1}{2}}}{\det(M_{c,i})^{\frac{1}{2}}} \frac{\tau_i^{\kappa_i}}{\Gamma(\kappa_i)} \frac{\Gamma(\kappa_i^*)}{(\tau_i^*)^{\kappa_i^*}} \qquad (41)$$

and further that

$$p(A|Y_T) \propto \frac{p(A)\det(A\hat{\Omega}A')^{\frac{T}{2}}}{\prod_{i=1}^{n}(\frac{2}{T}\tau_i^*)^{\kappa_i^*}} \prod_{i=1}^{n} \frac{\det(M_{c,i}^*)^{\frac{1}{2}}}{\det(M_{c,i})^{\frac{1}{2}}} \frac{\tau_i^{\kappa_i}}{\Gamma(\kappa_i)} \Gamma(\kappa_i^*) \qquad (42)$$

where we have multiplied equation (41) with $\prod_i^n (\frac{T}{2})^{\kappa_i^*} \det(\hat{\Omega})^{\frac{T}{2}}$, which does not affect our result in equation (42) since the term can be considered a constant w.r.t the elements of A. We have shown that the posterior of A can be sampled without sampling D and c. Furthermore, it is clear from the proof that if the posterior of A is known, we can calculate the posteriors of D and c. Thus, taking the log of the right hand side of the last equation, we get our target function less impact matrix priors

$$
\begin{aligned}
q(A) = \log p(A) + \frac{T}{2}\log[\det(A\hat{\Omega}A')] - \sum_i^n \kappa_i^* \log\left(\tau_i^* \frac{2}{T}\right) \\
+ \sum_{i=1}^n \kappa_i \log \tau_i + \sum_{i=1}^n \frac{1}{2}\left[\log\frac{|M_{c,i}^*|}{|M_{c,i}|} + \log\frac{\Gamma(\kappa_i^*)}{\Gamma(\kappa_i)}\right] \\
= \log p(A) + \frac{T}{2}\log[\det(A\hat{\Omega}A')] - \sum_i^n \left(\kappa_i + \frac{T}{2}\right)\log\left[\left(\tau_i + \frac{\zeta_i}{2}\right)\frac{2}{T}\right] \\
+ \sum_{i=1}^n \kappa_i \log \tau_i + \sum_{i=1}^n \frac{1}{2}\left[\log\frac{|M_{c,i}^*|}{|M_{c,i}|} + \log\frac{\Gamma(\kappa_i + \frac{T}{2})}{\Gamma(\kappa_i)}\right] \\
= \log p(A) + \frac{T}{2}\log[\det(A\hat{\Omega}A')] \\
- \sum_i^n \left(\kappa_i + \frac{T}{2}\right)\log\left[\left(\frac{2\tau_i}{T} + \frac{\zeta_i}{T}\right)\right] + \sum_{i=1}^n \kappa_i \log \tau_i \\
+ \sum_{i=1}^n \frac{1}{2}\left[\log\frac{|M_{c,i}^*|}{|M_{c,i}|} + \log\frac{\Gamma(\kappa_i + \frac{T}{2})}{\Gamma(\kappa_i)}\right] \qquad (43)
\end{aligned}
$$

Note that in our setup $M_{c,i}^*$, $M_{c,i}$,κ_i, and κ_i^* don't depend on A, and in that case the target function becomes

$$
q(A) = \log p(A) + \frac{T}{2}\log[\det(A\hat{\Omega}A')] \qquad (44)
$$

$$
- \sum_{i=1}^n (\kappa_i^* \log \tau_i^* - \kappa_i \log \tau_i) \qquad (45)
$$

7.2 Rewriting a linear model with Kronecker product and a linear constraint

Recall that applying the vec operator on a $n \times k$ matrix results in a $nk \times 1$ column vector comprised of stacked column vectors. The operator has the property that

$$
\text{vec}(CF) = (F' \otimes I_n)\text{vec}(C) = (I_l \otimes C)\text{vec}(F)
$$

where C is a $(n \times k)$ matrix and F is a $(k \times l)$ matrix. Stacking the vectors in the original DSM model:

$$
Az_t = Bx_t + u_t \qquad (46)
$$

we get

$$
AZ = BX + U
$$

where A is $(n \times n)$, Z is $(n \times T)$, B is $(n \times k)$, X is $(k \times T)$, and U is $(n \times T)$. Employing the Kronecker product we can thus write

$$(I_T \otimes A)\boldsymbol{z} = (X' \otimes I_n)\,\text{vec}(B) + \boldsymbol{u}$$

where $\boldsymbol{z} = \text{vec}(Z)$ and $\boldsymbol{u} = \text{vec}(U)$. Using the commutation matrix we lastly get

$$(I_T \otimes A)\boldsymbol{z} = (X' \otimes I_n)\,R_K c + \boldsymbol{u} \tag{47}$$

8 Table appendix

Table 1: A list of model variables.

Model variables	
Varible name	Description
y	Output gap
π	Inflation
r	Nominal interest rate
s	Log of the real exchange rate
π_F	Import price inflation
π_E	5 year break-even inflation expectations
y^*	Trade weighted foreign output gap
π^*	Trade weighted foreign inflation
r^*	Trade weighted foreign interest rates

Table 2: Prior distributions of parameters inducing contemporaneous effects.

Prior distributions of A	
Coefficients	Distributions
β_y	$t^+_{3,0.5}(0.35)$
γ_y	$t^+_{3,0.5}(0.5)$
θ_y	$t^+_{3,0.5}(0.2)$

(Continued on next page)

Table 2: (continued)

Prior distributions of A	
Coefficients	Distributions
α_π	$t^+_{3,0.5}(0.30)$
ζ_π	$t_{3,0.5}(0.00)$
η_π	$t_{3,0.5}(0.00)$
α_r	$t^+_{3,0.4}(0.50)$
β_r	$t^+_{3,0.4}(1.50)$
ρ	$\text{Beta}(15,10)$
β_s	$t^+_{3,0.5}(0.56)$
γ_s	$t^+_{3,0.5}(0.75)$
λ_s	$t^+_{3,0.5}(0.56)$
ν_s	$t^+_{3,0.5}(0.75)$
α_F	$t^+_{3,1}(0.3)$
δ_F	$t^+_{3,1}(0.5)$
λ_F	$t^+_{3,1}(0.5)$

Table 3: Prior distributions of elements of the impact matrix.

Prior distributions of $\overline{H}$	
Coefficients	Distributions
$\det(A)$	$\Upsilon_{3,2,20}(1)$
$\overline{h}_{1,2}$	$\Upsilon_{3,2,-20}(-0.1)$
$\overline{h}_{1,3}$	$\Upsilon_{3,2,-20}(-0.6)$
$\overline{h}_{1,7}$	$\Upsilon_{3,1,20}(0.55)$
$\overline{h}_{2,5}$	$\Upsilon_{3,1,20}(0.1)$
$\overline{h}_{3,3}$	$\Upsilon_{3,2,20}(0.7)$
$\overline{h}_{4,3}$	$\Upsilon_{3,2,-10}(-0.85)$
$\overline{h}_{5,8}$	$\Upsilon_{3,3,20}(0.23)$

Table 4: Posteriors of dynamic parameters

	Posterior	
	Mean	Stdev.
β_y	0.089	0.0534
γ_y	0.626	0.1105
θ_y	0.529	0.0651
α_π	0.134	0.0910
ζ_π	0.093	0.0113
η_π	0.070	0.1122
α_r	0.988	0.3052
β_r	1.119	0.2657
β_s	0.497	0.2899
γ_s	0.876	0.1596
λ_s	0.552	0.3083
ν_s	0.671	0.4483
α_F	0.775	0.4596
δ_F	1.007	0.1042
λ_F	0.857	0.3316
ρ	0.625	0.0808

Table 5: Average forecast variance decomposition contribution, over 8 years, in percent.

	Shocks								
	ε_y	ε_π	ε_r	ε_s	ε_{π_m}	ε_{π_E}	ε_{y^*}	ε_{π^*}	ε_{r^*}
π	4.2	9.4	0.8	13.1	6.3	2.0	49.2	13.6	1.5
y	39.2	6.0	5.5	21.3	6.3	8.1	7.4	5.4	0.9
r	11.9	2.0	5.8	1.9	1.1	11.9	61.5	2.5	1.5

9 Figure appendix

9.1 Prior and posterior densities

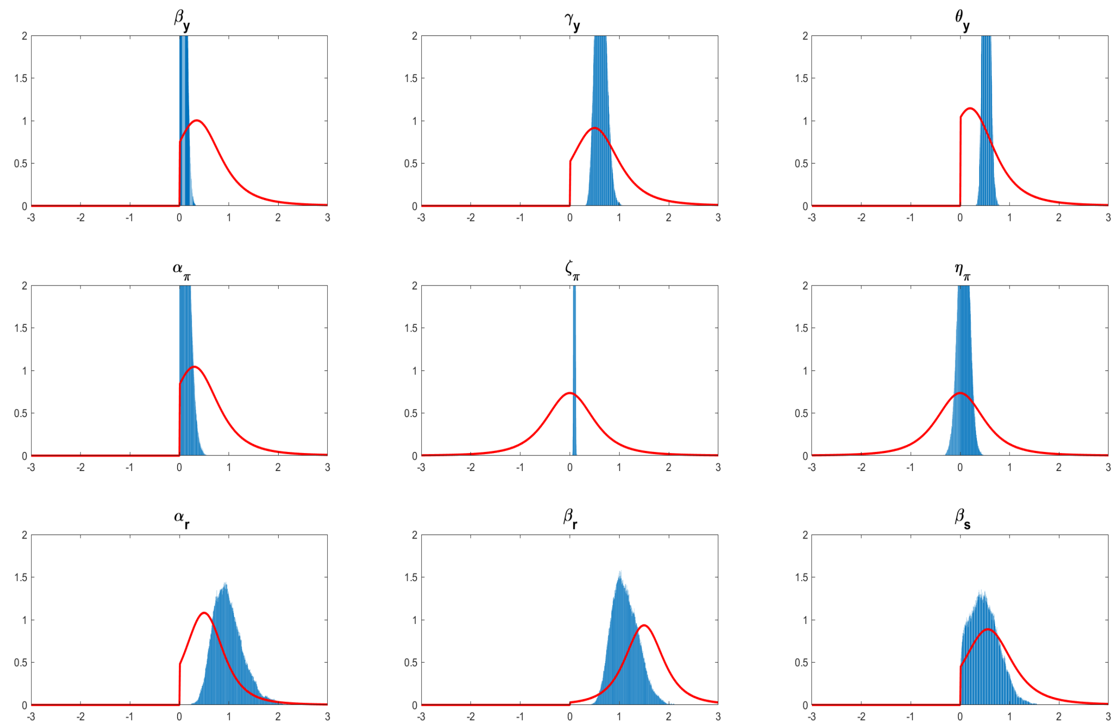

Figure 1: Prior and posterior densities of contemporaneous parameters. The red line represents the prior density and the blue area the posterior density.

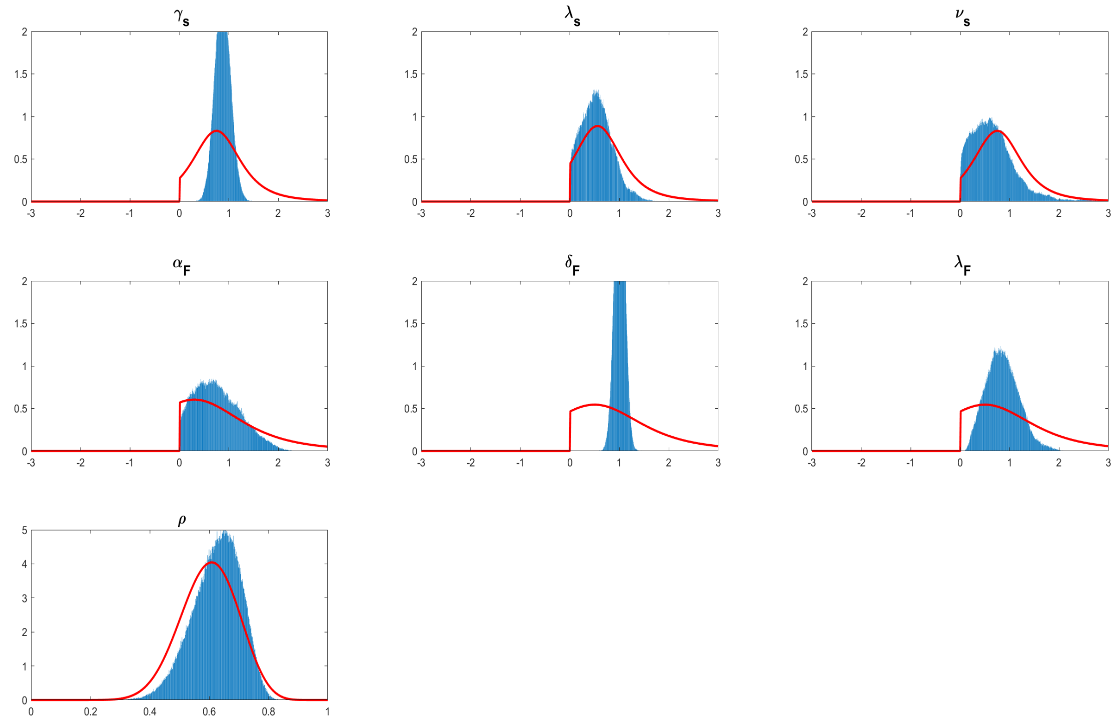

Figure 2: Prior and posterior densities of contemporaneous parameters. The red line represents the prior density and the blue area the posterior density.

9.2 Impulse response functions

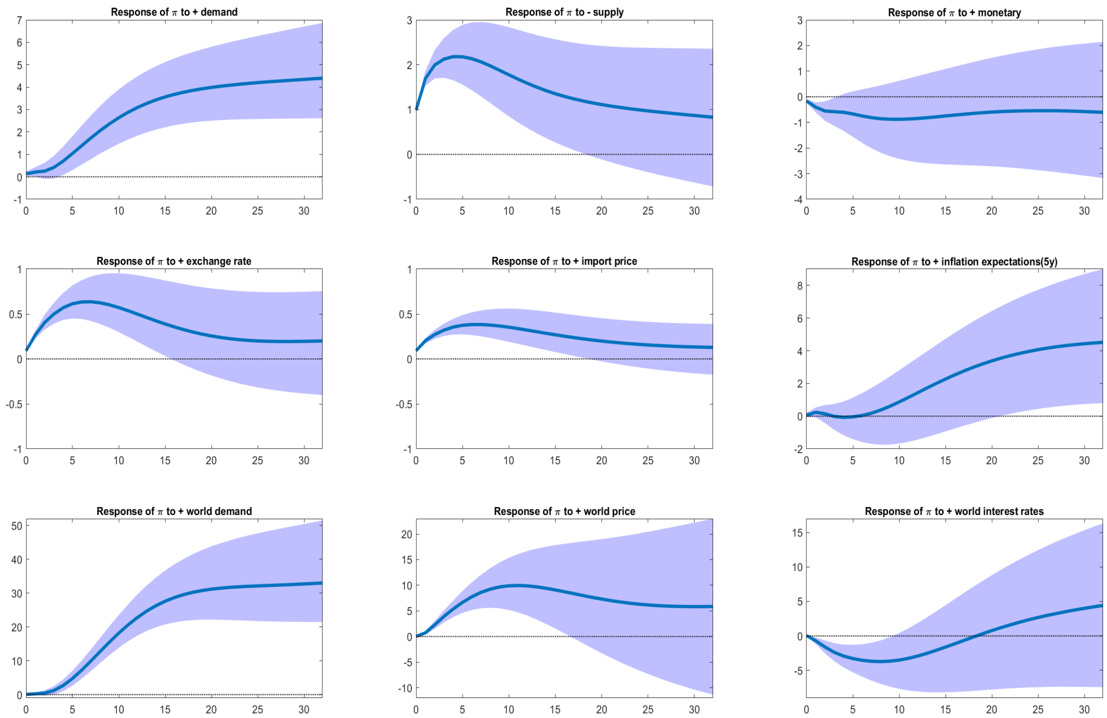

Figure 3: Cumulative impulse response functions of inflation. Blue solid line is the median and the shaded area is the 68% credibility interval.

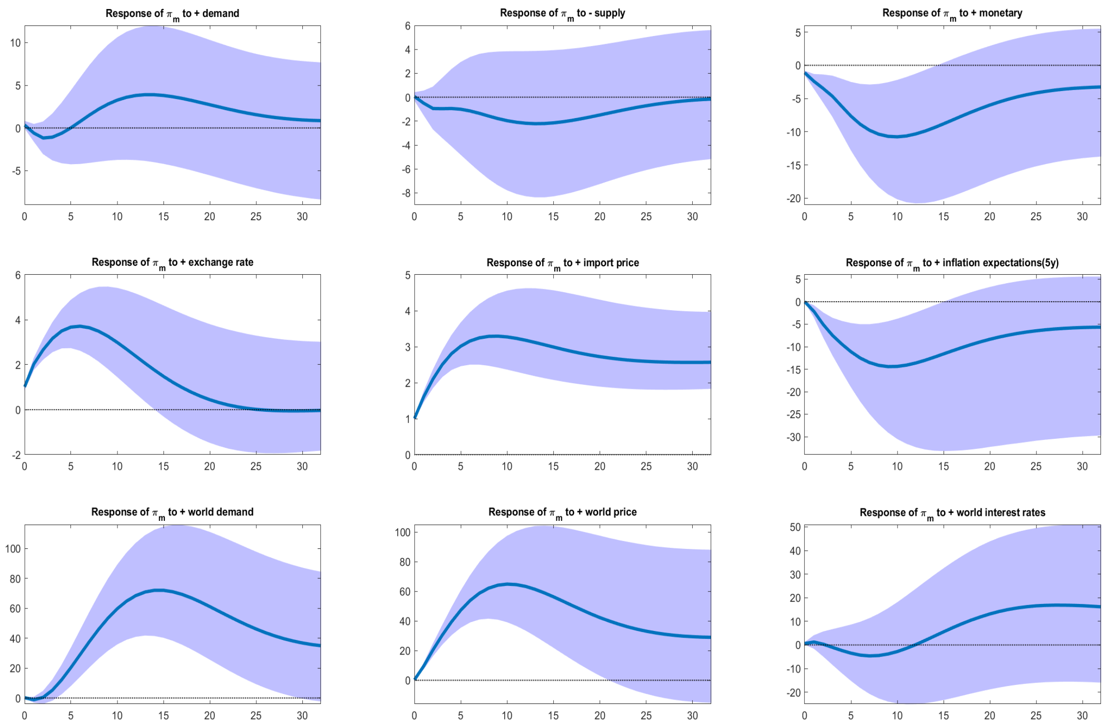

Figure 4: Cumulative impulse response functions of import inflation. Blue solid line is the median and the shaded area is the 68% credibility interval.

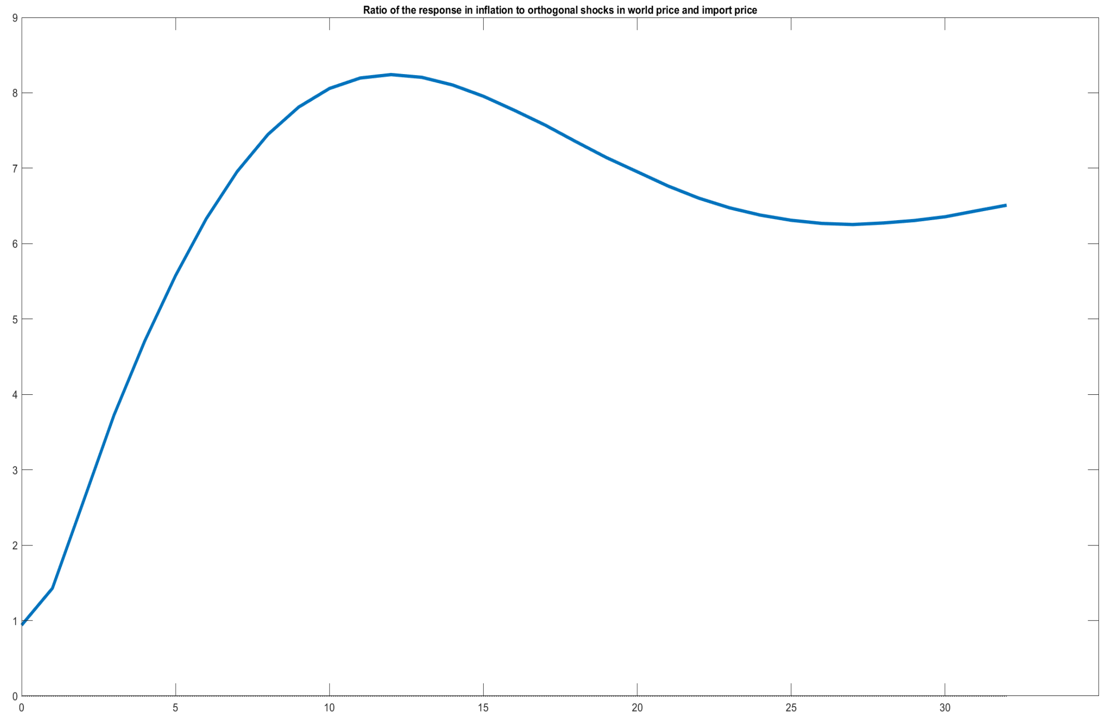

Figure 5: Ratio of the median impulse responses of inflation to π^* and π_m.

9.3 Forecast variance Decomposition

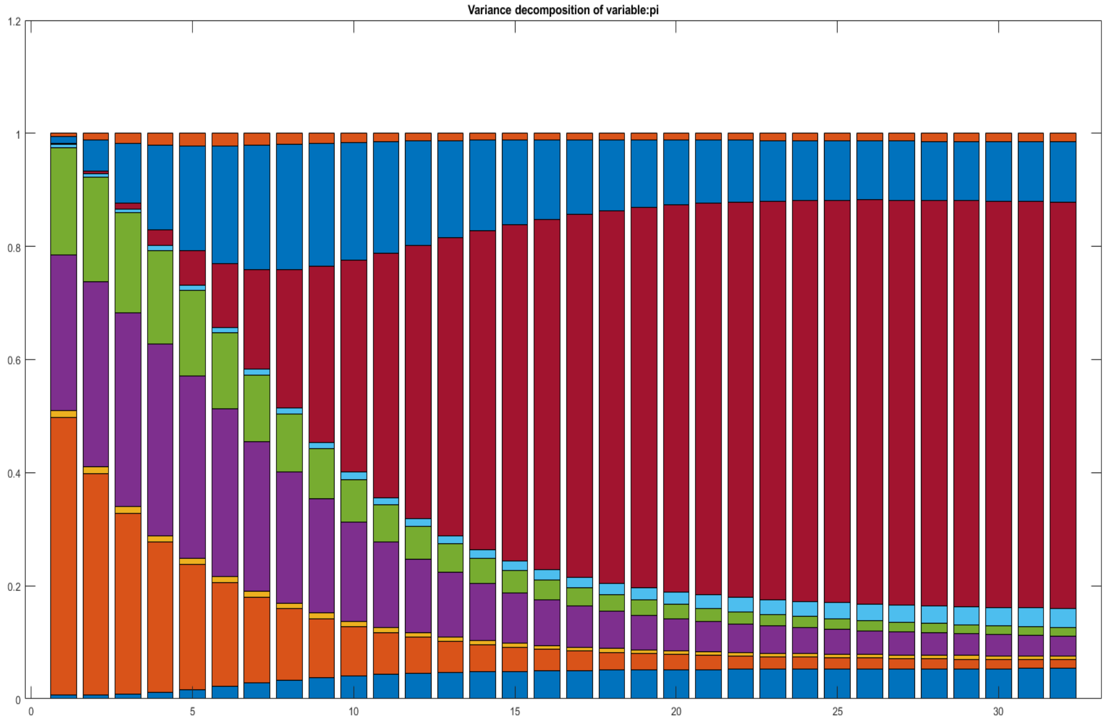

Figure 6: Forecast variance decomposition of inflation. Blue is a demand shock, orange is a supply shock, yellow is an interest rate shock, purple is an exchange rate shock, green is import inflation shock, light blue is inflation expectations shock, red is a foreign demand shock, dark blue is foreign price shock, dark orange is a foreign interest rate shock.

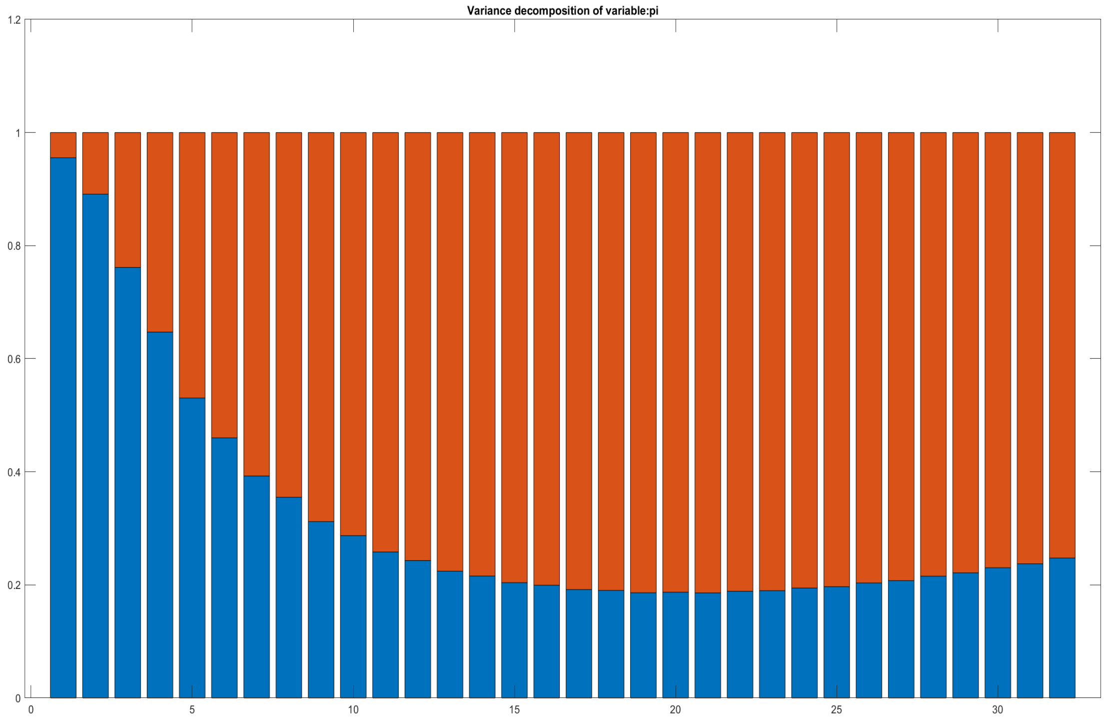

Figure 7: Forecast variance decomposition of inflation 2011Q1:2019:Q4. Blue columns indicate domestic factors. Orange columns are foreign factors.

9.4 Historical variance decomposition

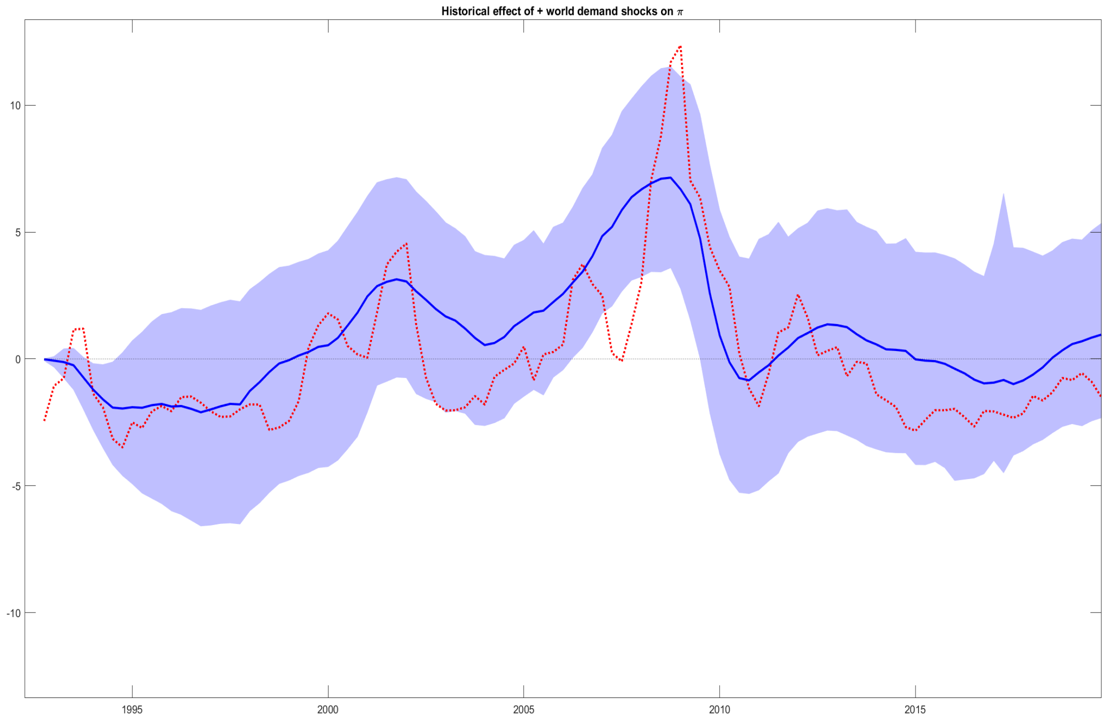

Figure 8: Red dashed line is actual values of inflation gap from it's mean. Blue solid line is the estimated contribution of world demand. Shaded area is the 68% posterior credibility region.

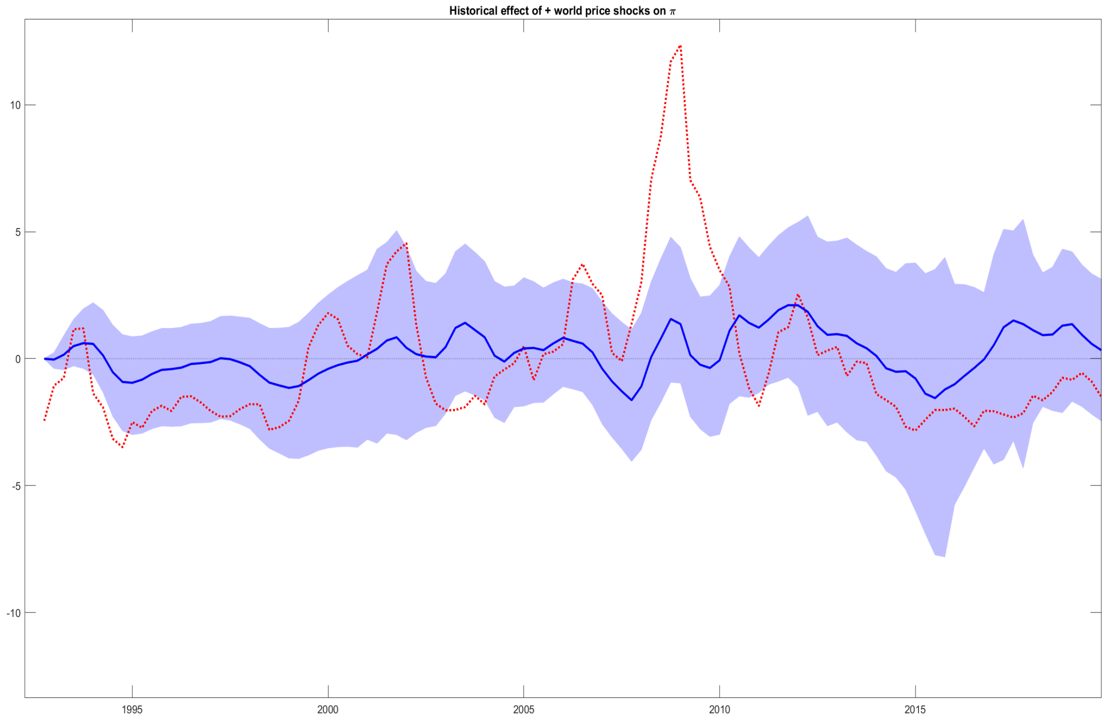

Figure 9: Red dashed line is actual values of inflation gap from it's mean. Blue solid line is the estimated contribution of world price. Shaded area is the 68% posterior credibility region.

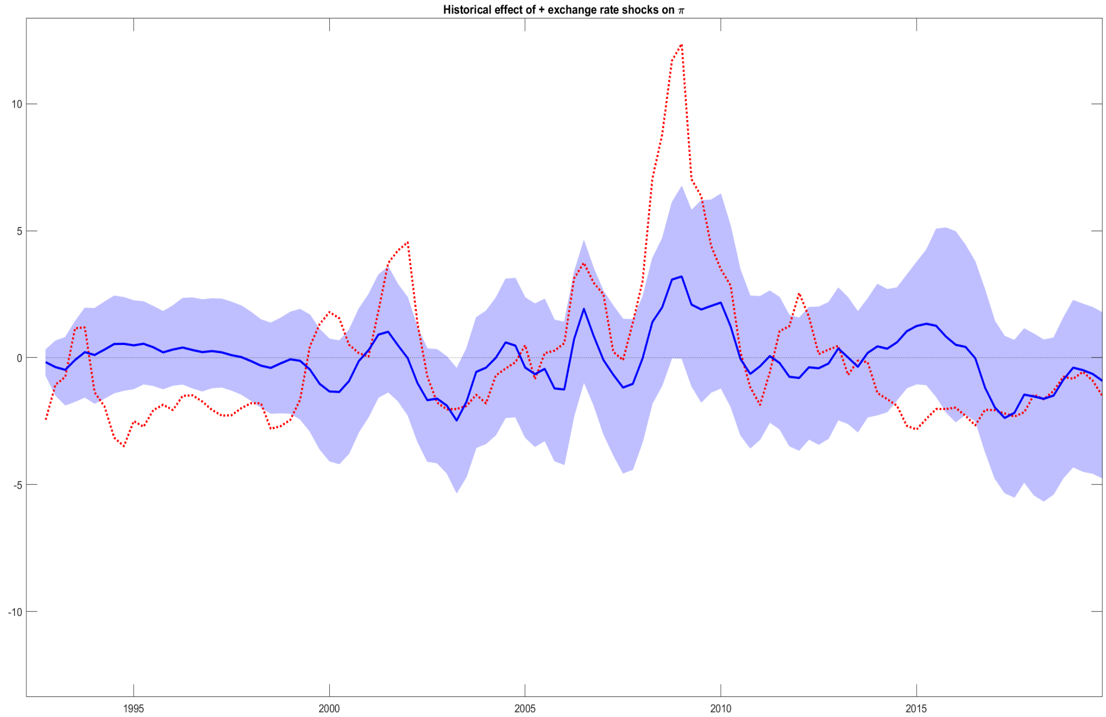

Figure 10: Red dashed line is actual values of inflation gap from it's mean. Blue solid line is the estimated contribution of exchange rates. Shaded area is the 68% posterior credibility region.

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