WORKING PAPER

CENTRAL BANK OF ICELAND

No. 74

Estimating the Natural Interest Rate for Iceland: An Exploratory Study

By

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December 2016

Central Bank of Iceland Working Papers are published by the Economics and Monetary Policy Department of the Central Bank of Iceland. The views expressed in them are those of their authors and not necessarily the views of the Central Bank of Icleand.

Also available on the Central Bank of Iceland World Wide Web site (http://www.sedlabanki.is)

ISSN 1028-9445

Estimating the Natural Interest Rate for Iceland: An Exploratory Study*

Ásgeir Daníelsson[†], Ólafur Sindri Helgason[†], Stefán Thórarinsson[†]

December 5, 2016

Abstract

We estimate the natural rate of interest for Iceland using several different methods. First we explore estimates based on the marginal productivity condition for capital, continue with various estimates based on the Euler equation for optimizing households, and end with estimates using state-space models where the natural rate is an unobservable variable, including the celebrated model by Laubach and Williams. Some of these methods give unreasonable estimates while others provide estimates that should be helpful for policy makers. The state-space models give estimates of the natural rate with negative trend before and immediately after the financial crises, but recovering during last four years of fairly rapid economic growth. The paper also provides estimates of the monetary policy stance in Iceland during the last 20 years and estimates of the implicit inflation target.

Keywords: Equilibrium real rate, Euler equations, state space models, Bayesian methods, Iceland

JEL Classification: C32, C51, E44, E58

*We thank Aðalheiður Ósk Guðlaugsdóttir, Thórarinn G. Pétursson, Már Gudmundsson and seminar participants at CBI on 29 November 2016. All errors and omissions are our own.

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1 Introduction

The natural, or equilibrium, rate of interest is a central concept in modern monetary theory. Its origin can be traced back to the Swedish economist Knut Wicksell, who described in his book Interest and prices, first published in 1898, a mechanism by which movements in the rate of interest affect the price level. If the rate at which the banks lend to the firms is below the equilibrium level, firms are profitable and try to expand their operations leading to excess demand for the factors of production, resulting in increases in their prices, and after that, through a rise in demand for consumption goods, increases in the general price level. If, on the other hand, the rate of interest is above the equilibrium rate, firms are making losses, demand for factors of production diminish, and eventually demand for goods and services declines and so does the price level. In Wicksell's view the authorities didn't need to measure the equilibrium rate of interest. It would be sufficient to watch movements in the price level. If the price level moves up it indicates that the rate of interest is below the equilibrium rate and the authorities should increase it to lower the price level, and if the price level moves down, the interest rate should be lowered below the equilibrium rate to restore the price level to its previous level.

In modern monetary theory there are more channels through which monetary policy affects the price level and the rate of inflation, especially in open economies. It also recognizes expectations as a special factor which can move the price level by itself but can also be influenced by monetary policy.

The interest in estimating the natural rate of interest has increased in recent years as a number of researchers have noted that the rate of interest has been declining in Western countries during last $20 - 30$ years. Some researchers[1] argue that the Western countries have entered a period of secular stagnation where the natural rate of interest is likely to remain low making it difficult for monetary policy to create sufficient stimulus for economic growth because of the zero lower bound on the nominal interest rate. In most Western countries, and especially in Japan, inflation has remained low in spite of considerable efforts by central banks to increase it by reducing their policy rates to zero, or even a bit below zero in some cases, and by increasing the money supply through large purchases of financial assets (the so called quantitative easing).

The Icelandic economy behaved differently compared to these economies dur-

[1] See e.g. Summers (2014) and Eggertsson et al. (2016).

ing the financial crisis as inflation increased, and peaked at 18.7% in January 2009. Inflation has subsided since then and remained below the inflation target of the Central Bank of Iceland (CBI) since early 2014. However inflation expectations have, until very recently, remained similar to what they were before the crisis of 2008, and were consistently above the inflation target of the CBI. Consequently, interest rates in Iceland have been higher than in most OECD countries, and, after a sharp downturn in 2009 and 2010, economic growth in Iceland has also been higher than in most OECD countries.

Figure 1 and 2 show the development of long run real interest rates in some OECD countries, including Iceland. The rate in a given month is calculated using nominal interest rate in the given month while the expected rate of inflation is estimated as the average rate of inflation during 12 months prior to the month and 12 months after. This choice is obviously somewhat arbitrary but the main trends are robust against alternative definitions. Iceland is different from the other countries shown in these figures in that inflation has been higher, and more variable, making calculations of real interest rates less accurate, whichever method is used to approximate inflation expectations. But, for these same reasons, Iceland is also different in that large part of the bond market consists of inflation protected government bonds (TIPS). For Iceland Figure 2 shows data from the CBI on 10 year inflation protected government bonds.

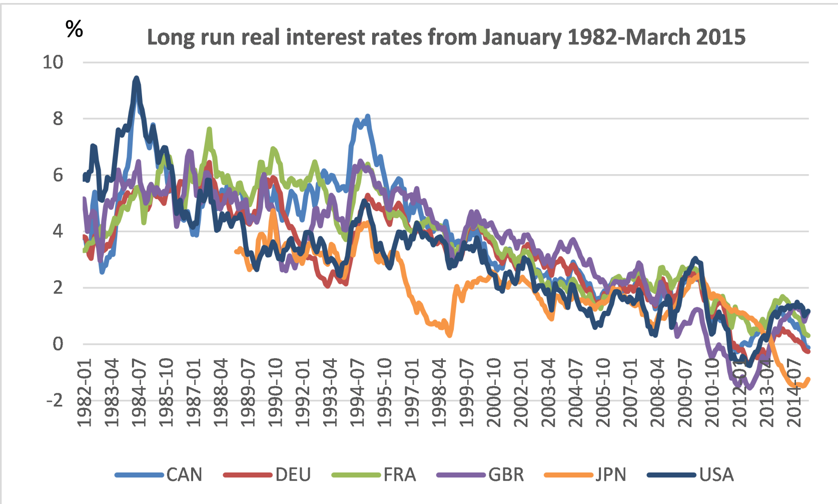

Figure 1. Sources: OECD database - https://data.oecd.org/interest/long-term-interest-rates.htm, and calculations by the authors, and CBI.

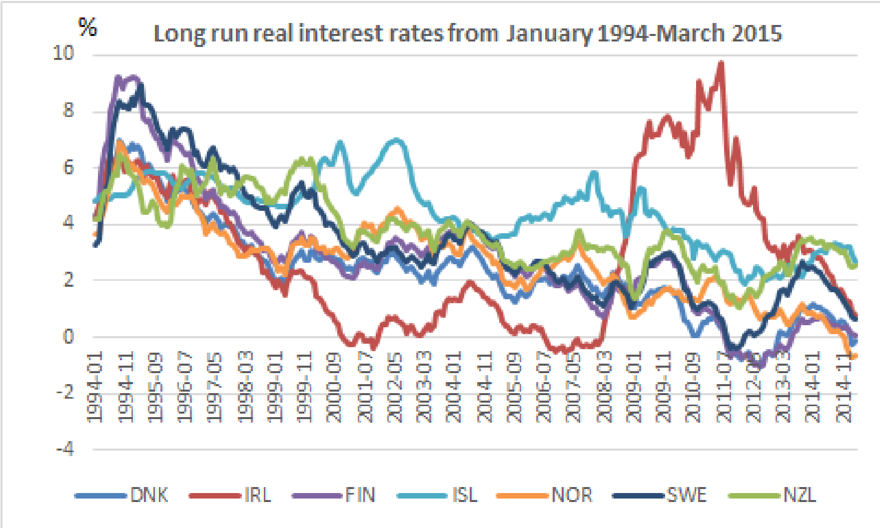

Figure 2. Sources: OECD database - https://data.oecd.org/interest/long-term-interest-rates.htm, and calculations by the authors, and CBI.

Testing for linear trends in the data show that there is clear negative trend in most cases. For the large economies like US, UK, Germany and France we find a significant negative trend both when the estimation period is from January 1982 to March 2015, and when it is from January 1982 to December 2006, i.e. when we exclude the period of the financial crises and its aftermath. The same is true for Canada, but the data for Japan only start in January 1989.

For the countries in Figure 2 we find that estimations for the period from January 1994 to March 2015 give negative linear trend, except for Ireland where there is no trend over this period. In Ireland's case there is clear negative trend over the period from January 1994 to December 2006. In Iceland's case the estimated trend is negative and significant in both cases in spite of the sharp increase since February 2012.

In this paper we will discuss three main methods to estimate the equilibrium real rate, and present estimates using Icelandic data. In Section 2 we will discuss estimation using the condition from the profit-maximization of firms, which says that the cost of capital should be equal to the marginal product of capital. In Section 3 we will discuss various estimates based on the Euler equation which is derived from the households' objective to maximize utility over time. In Sections 4 and 5 the equilibrium real rate of interest is modelled as an unobservable variable in a state-space model. The predictions of the model are used to infer an estimate of the equilibrium rate. If the model predicts inflation above the actual inflation it may indicate that the estimate of the natural rate is too low. Similarly, the opposite conclusion follows if actual inflation is below the prediction of the model. The equilibrium natural rate is estimated together with the parameters of the model. In Section 4 we will discuss estimations based on two simple backward-looking state-space models which are estimated with maximum likelihood methods via the Kalman filter. The first model is the closed economy model by Laubach and Williams (2003) which has been used by the Federal Reserve in the US and by the European Central Bank. The second model in Section 4 is the open economy model by Kirker (2008). In Section 5 a forward-looking state-space model is estimated with Bayesian methods. The models in Sections 4 and 5 provide short-term, time-varying estimates of the equilibrium real rate. Section 6 discusses and compares the results from the estimations in Sections 4 and 5 and Section 7 provides conclusions for all sections of the paper, and discusses some aspects of their use in monetary policy.

2 Estimation based on the marginal product of capital

If the production function is of Cobb-Douglas type, i.e.

$$Y_t = A_t K_{t-1}^{1-\alpha} L_t^{\alpha} \tag{1}$$

where Y_t is output, K_{t-1} is the capital available at the end of period $(t-1)$, L_t is labour used in the production and A_t is exogenous productivity, and the firms' objective is to maximize profit:

$$\pi_t = P_t Y_t - W_t L_t - (r + \delta) P_t^K K_{t-1} \tag{2}$$

where π_t is profit, r is the real rate of interest, δ is the rate of depreciation, and P_t^K is the price of capital, then the maximization condition for Equation (2) is:

$$\frac{\partial \pi_t}{\partial K_{t-1}} = (1-\alpha) P_t \frac{Y_t}{K_{t-1}} - (r+\delta) P_t^K = 0$$

$$\Leftrightarrow r = (1-\alpha) \frac{P_t Y_t}{P_t^K K_{t-1}} - \delta = \frac{1-\alpha}{P_t^K K_{t-1} / (P_t Y_t)} - \delta \tag{3}$$

The coefficient α is estimated by the share of wage cost in gross factor income which means that the numerator in the rightmost expression in Equation (3) is one minus the share of wage cost, and the denominator is the capital-output ratio. The equation predicts that the real rate of interest is equal to this ratio minus the rate of depreciation[2]. An increase in the share of wage cost will lower the equilibrium rate of interest. An increase in the capital-output ratio will also depress the equilibrium rate of interest. The same will happen if the rate of depreciation increases.

The rate of depreciation has been stable for a long time in Iceland, close to 0.04 for the total economy and around 0.06 if we exclude the public sector and housing. The capital output ratio has also been fairly stable around 3.3 for the total economy and 1.9, excluding the public sector and housing. The deviations of the capital output ratio from the average value are though slightly larger than in the case of the rate of depreciation. Figure 3 shows the capital output ratios.

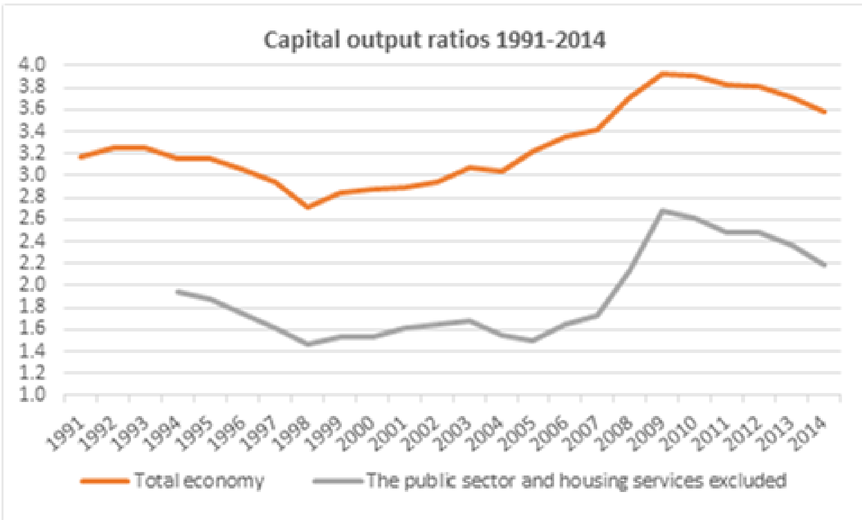

Figure 3. Source: Statistics Iceland

The increases in the capital output ratios during the years before, and immediately after, the collapse of the banks in 2008 is explained by the combination of large investments in fixed capital during the years before the crisis, and a

[2]It is easy to show that this relationship holds also when the production function is of CES-type (Constant Elasticity of Substitution) which is more general than the Cobb-Douglas function.

sharp decline in output during its aftermath.

The national accounts classify all income of self-employed workers as operating surplus rather than wage. In the income accounts there is an item called "calculated owners income" (Icelandic: "Reiknuð laun vegna atvinnureksturs"). If the share of wage cost in total factor income is adjusted for this item Figure 4 emerges.

Figure 4. Source: Statistics Iceland

Using this data on depreciations, capital output ratio, and the share of wage cost, Equation (3) provides the results shown in Figure 5. In the figure the equilibrium real rate of interest has been calculated using fixed average shares of wage costs of 0.7, 0.65, and 0.6, and also by using the estimated share of wage cost in the same year. All estimates indicate that the real rate has been declining over this period, mainly due to increases in the share of wage costs but also because of increases in the capital output ratio.

The analysis in this section ignores the fact that there are risks involved in all economic activities. It also ignores other capital assets than the fixed capital which is registered in the national accounts and calculated using the Perpetual Inventory Method (PIM). This means that the capital value of natural capital like land and fish stocks, which are very important for the Icelandic economy, are not included, nor the capital value of intangible assets. Including these items would lower the estimates of the equilibrium real rate of interest compared to what is shown in Figure 5.

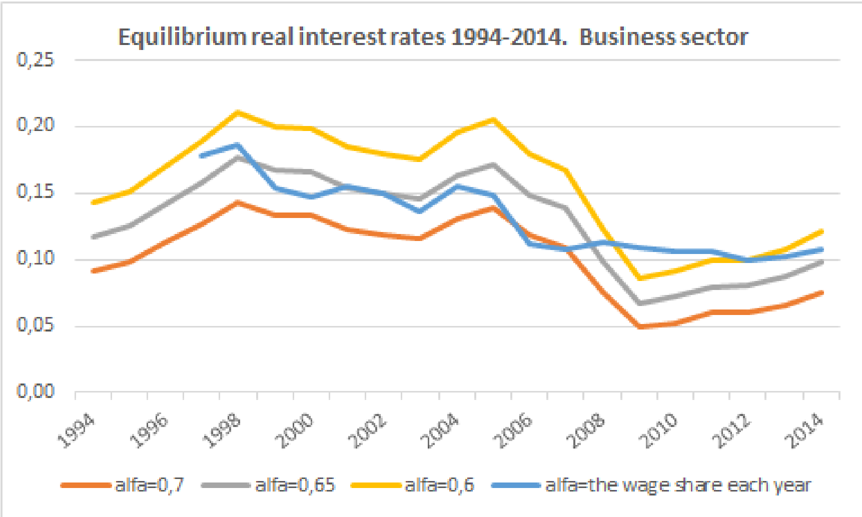

Figure 5. Natural real rates of interest derived from the marginal product of capital.

It is worth noting here that the share of wage cost has developed somewhat differently in Iceland compared to almost all other economies within the OECD. The OECD Employment Outlook (OECD, 2012) discusses the fact that almost all OECD countries have experienced significant downward trend in the wage share.

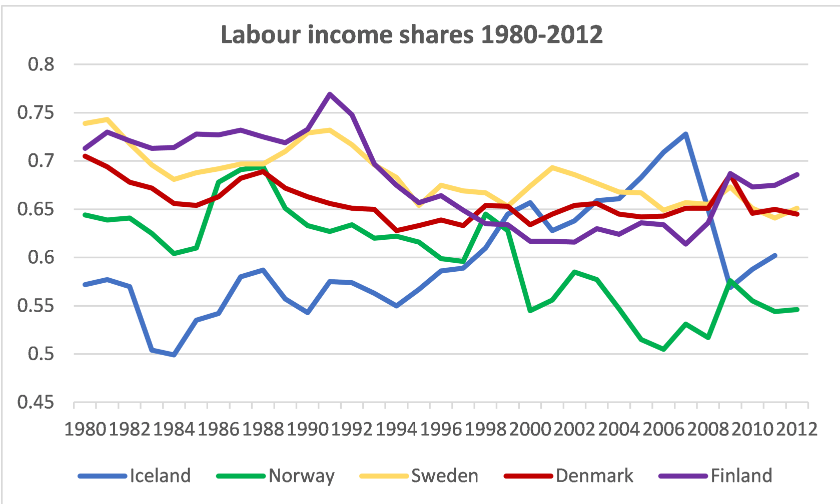

Figure 6. Wage shares for five Nordic countries. Source: OECD database, Unit labour cost-Annual indicators.

Figure 6 shows that the wage share has declined in all the Nordic countries, except Iceland. This means, of course, that the development of the share of wage cost cannot explain some part of the decline in the equilibrium real rate of interest in those countries as it can in Iceland. It is also interesting to note that while the immediate reaction to the financial crisis that hit in 2008 was a sharp decline in the wage share in Iceland, the wage share in the other Nordic countries actually increased.

3 Estimations using the Euler equation

The local condition for optimal consumption over time in deterministic setting is given by the Euler equation:

$$U'(C_t) = \beta \left(\frac{1 + R_t}{\Pi_{t+1}} \right) U'(C_{t+1}) \tag{4}$$

where U' is the derivative of the utility function, R_t is the nominal rate of interest in period t which is to be paid in period $t+1$, $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the gross rate of inflation, and P_t is the price level so that $\frac{1+R_t}{\Pi_{t+1}}$ is the gross real rate of interest between periods t and $t+1$. Further, β is the discount factor reflecting that households discount future utility, and C_t is consumption. It follows that if the household saves the amount of money that can buy one unit of consumption goods in period t, it will be able to buy $\frac{1+R_t}{\Pi_{t+1}}$ units in period $t+1$. In normal circumstances the real rate of interest would be positive, and therefore the household will be able to increase the volume of consumption goods by saving today and spending later. But, at the same time, the household is impatient and values consumption in the future less than the same consumption today, as reflected by the factor $\beta < 1$.

It is easy to see the rationale for the condition in Equation (4). If the left hand side is larger than the right hand side the utility from one additional consumption unit in the present period t is larger than the loss of utility from decreasing consumption in period $t+1$ by $\frac{1+R_t}{\Pi_{t+1}}$ units of consumption goods that are discounted by the factor β. It therefore increases total utility to increase consumption in the present period. And, if the left hand side of Equation (4) is smaller than the right hand side, then it increases total utility to decrease consumption in the present period (increasing the marginal utility in this period) to be able to increase consumption by $\frac{1+R_t}{\Pi_{t+1}}$ units in the next period. It follows that Equation (4) is an optimality condition.

If the stochastic nature of some of the variables is taken into account the Euler equation becomes:

$$U'(C_t) = \beta(1+R_t) E_t \left[U'(C_{t+1}) \Pi_{t+1}^{-1}\right] \tag{5}$$

where E_t is the expectation operator using information available in period t. In Equation (5) consumption, C_t, and therefore $U'(C_t)$ as well as R_t and P_t are assumed to be known in period t, and therefore not stochastic variables, but C_{t+1} and P_{t+1} are stochastic variables. The discount rate, β, is assumed to be non-stochastic and constant over time.

Most researchers assume that the utility function is of constant relative risk aversion (CRRA) type, i.e.

$$U(C_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma}$$

where σ is the coefficient of relative risk aversion. For $\sigma = 1$ this utility function becomes $U(C_t) = \log(C_t)$. Substituting the CRRA utility function into Equation (5) gives that

$$C_t^{-\sigma} = \beta(1+R_t) E_t \left\{\frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}}\right\} \tag{6}$$

First order Taylor expansion around $\frac{C_{t+1}}{C_t} = 1$ and $\Pi_{t+1} = 1$ gives:

$$1 = \beta(1+R_t) E_t \left\{1 - \sigma\left(\frac{C_{t+1}}{C_t} - 1\right) - (\Pi_{t+1} - 1) + O\left(\Pi_{t+1}^2 + \left(\frac{C_{t+1}}{C_t}\right)^2\right)\right\}$$

This can be simplified to give:

$$R_t - E_t\{\pi_{t+1}\} \approx \sigma E_t \left\{\frac{C_{t+1}}{C_t} - 1\right\} + (1-\beta) \tag{7}$$

where $\pi_{t+1} = \Pi_{t+1} - 1$ is the rate of inflation.

If $U(C_t) = \log(C_t)$, and therefore $\sigma = 1$, this equation states that the equilibrium real rate of interest is equal to the sum of expected economic growth per household in the next period (or growth in consumption per person), and $(1-\beta)$ which is equal to the discount rate used to discount future utility. If $\beta = 0.9915$, as it is presently in DYNIMO[3] the annual discount rate is

[3]CBI quarterly DSGE model. See Seneca (2010) for a model description

$0.9915^4 = 0.9664$. If consumption per capita is growing at 1.66% per year, which is the assumption for long run growth of productivity in QMM, CBI's quarterly macro model[4], Equation (7) gives the equilibrium real interest rate:

$$R_t - E_t\left\{\pi_{t+1}\right\} = \sigma E_t\left\{\frac{C_{t+1}}{C_t} - 1\right\} + (1-\beta) = 0.0166 + 1 - 0.9664 = 0.050$$

There are several problems with using this equation to determine the equilibrium real rate. One is that the subjective discount factor is a bit arbitrary. Some writers (see e.g. Fisher (1930)) have used the real rate of interest to measure the subjective discount factor. In that case the Euler equation gives that the rate of growth in consumption that maximizes utility of risk-averse consumers is zero, i.e. the consumer maximizes his/hers utility when the consumption is the same in every period. Armelius et al. (2014) estimate the equilibrium real rate of interest for Sweden assuming that $\beta = 1$, in which case the linear Euler equation for logarithmic utility gives that the equilibrium real rate of interest is equal to the growth of per capita consumption. Another problem is that it is difficult to obtain reliable estimates of the relative risk aversion. Lucas (1987) considers relative risk aversion of unity, which is assumed above by assuming logarithmic utility, too low and considers values even above 10.[5] Others have estimated lower values, e.g. Pétursson (2000) who estimates the relative risk aversion in Iceland to be 0.25.

Table 1 gives the equilibrium real rate of interest (r^*) according to Equation (7) above if the private consumption per capita is growing at the rate of 1.66%, the subjective discount factor $\beta = 0.9664$, and the coefficient of relative risk aversion (σ) is as shown in the top row, which spans the range of values for this coefficient that are considered realistic.

σ	0.25	0.5	1	2	5	10
r^*	0.038	0.042	0.050	0.067	0.117	0.200

Table 1. Natural real interest rates as a function of relative risk aversion

[4]See the database for QMM on the CBI website http://www.sedlabanki.is/peningastefna/efnahagsspa/. The variable TpGDPT gives the logarithm of the gross growth in productivity per quarter, which is equal to the quarterly growth of output per capita if the age composition of the population remains constant.

[5]See Mehra and Prescott (1985)

3.1 Habit persistence but no risk

The utility function in DYNIMO is separable in consumption and labour. The consumption part is:

$$U\left(C_t\left(j\right)\right) = Z_{C,t}\log\left(C_t\left(j\right) - hC_{t-1}\right)$$

where C_{t-1} is the aggregate consumption per capita in the previous period (i.e. not part of the household's j's decision problem), $Z_{C,t}$ are taste shocks and h is the coefficient of habit persistence. The coefficient of relative risk aversion becomes in this case:

$$\rho = \frac{C_t\left(j\right)}{C_t\left(j\right) - hC_{t-1}} > 1$$

when $h > 0$. If $h = 0.8$, which is close to the estimates obtained in DYNIMO then $\rho \approx 5$, i.e. allowing for habits persistence in consumption increases the relative risk aversion fivefold compared to the baseline model where there is no habit formation and $h = 0$.

Abstracting from problems caused by risk and stochastic variables the Euler equation for this case can be obtained by differentiating the utility function and substitute the derivatives into Equation (5) to obtain

$$\frac{1+R_t}{\Pi_{t+1}} = \frac{1}{\beta}\frac{C_{t+1}\left(j\right) - hC_t}{C_t\left(j\right) - hC_{t-1}}$$

On a constant growth path where the per capita consumption grows at the constant rate of g, i.e $C_{t+1}(j) = (1+g)C_t(j)$, we find that

$$\frac{C_{t+1}\left(j\right) - hC_t}{C_t\left(j\right) - hC_{t-1}} = (1+g)\frac{C_t\left(j\right) - hC_{t-1}}{C_t\left(j\right) - hC_{t-1}} = \frac{C_{t+1}\left(j\right)}{C_t\left(j\right)}$$

and we obtain the same results concerning the equilibrium real rate as in the case where utility is logarithmic, and there is no habit persistence.

3.2 Risk but no habit persistence

Carroll (2001) investigates the relevance of simplifications of the Euler equation when the utility function is CRRA, i.e. he analysis Equation (6). He calculates the optimal consumption paths for a number of imaginary households with known utility functions, and then uses these "data" to estimate the Euler equation using the linear approximation of the true equation. Carroll finds that he is not able to estimate the deep parameters from the "data" using the linear version of the Euler equation above. From which it is concluded that second order, and even higher order terms in the Taylor expansion, matter for the estimation of the true parameters of the utility functions.

Deriving the second order Taylor expansion for C_{t+1}/C_t, while keeping the first order expansion around Π_{t+1}, and omitting higher order terms gives the following version of the Euler equation:

$$1 \approx \beta (1 + R_t) E_t \left\{ 1 - \sigma \left(\frac{C_{t+1}}{C_t} - 1 \right) + \frac{\sigma (\sigma + 1)}{2} \left(\frac{C_{t+1}}{C_t} - 1 \right)^2 - \pi_{t+1} \right\}$$

Similarly as above, we can approximate the Taylor expansion and get:

$$R_t - E_t \{\pi_{t+1}\} \approx \sigma E_t \left\{ \frac{C_{t+1}}{C_t} - 1 \right\} - \frac{\sigma (\sigma + 1)}{2} E_t \left\{ \left(\frac{C_{t+1}}{C_t} - 1 \right)^2 \right\} + (1 - \beta)$$

The term $\frac{\sigma(\sigma+1)}{2} E_t \left\{ \left(\frac{C_{t+1}}{C_t} - 1 \right)^2 \right\}$ can be interpreted as a premium based on the risk in consumption. Estimating, using annual data for Iceland from 1990 to 2014, gives $E_t \left\{ \left(\frac{C_{t+1}}{C_t} - 1 \right)^2 \right\} = 0.0028$. Calculating the equilibrium real rate of interest on the basis of this formula gives the numbers in Table 2.

σ	0.25	0.5	1	2	5	10
r^*	0.037	0.041	0.047	0.058	0.075	0.045

Table 2. Natural real interest rates as a function of relative risk aversion based on second order Taylor expansion of the Euler equation

Comparing the results in Table 2 to those in Table 1 shows that the risk premium is very small when risk aversion is low (1 or 2) but increases rapidly when the coefficient of relative risk aversion increases.

3.3 Habit persistence and risk

If habit persistence is included, and risk is taken into account, the Euler Equation (6) for the utility function in Dynimo becomes:

$$1=\beta\left(1+R_t\right)E_t\left\{\left(\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}\right)^{-1}\frac{1}{\Pi_{t+1}}\right\}$$

Second order Taylor expansion of the right hand side around $\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}=1$ and a first order expansion around $\Pi_{t+1}=1$ gives that

$$1\approx\beta\left(1+R_t\right)E_t\left\{1-\left(\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}-1\right)+\left(\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}-1\right)^2-\pi_{t+1}\right\}$$

Repeating the earlier process, we can approximate the Taylor expansion and get:

$$R_t-E_t\left\{\Pi_{t+1}-1\right\}\approx E_t\left\{\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}-1\right\}-E_t\left\{\left(\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}-1\right)^2\right\}+(1-\beta)$$

The term $E_t\left\{\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}-1\right\}$ can be estimated by the average growth of per capita consumption as in the case where there is no habit persistence, but the term $E_t\left\{\left(\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}-1\right)^2\right\}$ is larger than $E_t\left\{\left(\frac{C_{t+1}}{C_t}-1\right)^2\right\}$ and if h is as high as 0.8 (which is close to the number used in DYNIMO) then $E_t\left\{\left(\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}-1\right)^2\right\}$ is much larger than $E_t\left\{\left(\frac{C_{t+1}}{C_t}-1\right)^2\right\}$.

Above we estimated $E_t\left\{\left(\frac{C_{t+1}}{C_t}-1\right)^2\right\}$ from annual data for consumption per capita in Iceland to be 0.0028. Estimating $E_t\left\{\left(\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}-1\right)^2\right\}$ where $h=0.8$ using the same data gives an estimate of 0.0669. The risk premium in this case is so large that the equilibrium real rate, given by the Euler equation, becomes negative even at moderate levels of risk aversion, as is shown in Table 3.

σ	0.25	0.5	1	2	5	10
r^*	0.027	0.017	-0.017	-0.134	-0.886	-3.478

Table 3. Natural real interest rates as a function of relative risk aversion based on second order Taylor expansion of the Euler equation with habit persistence.

In recent papers, McKay et al. (2016a) and McKay et al. (2016b), it is pointed out that the linear Euler equation that is commonly used in New Keynesian DSGE models leads to unreasonably large effects of forward guidance in the form of central bank declaration of a interest rate change in some distant future period of time. In Mckay et al. (2016a) it is suggested that the standard Euler equation should be replaced by what the authors call "the discounted Euler equation". Using this equation to calculate the equilibrium real rate gives lower estimates than the standard version, and sometimes negative rates.

4 Empirical investigation using backward-looking models

It is possible to model the natural rate of interest as an unobservable variable in a state space model, and infer its value from the effects it has on observable variables. To accomplish this we use two approaches, the first of which is due to Laubach and Williams (2003), who use the Euler equation from intertemporal maximization to express the natural interest rate as a linear function of growth of the output potential, and a composite w_t, which encompasses time preference along with a risk premium, and other unspecified factors. We can thus write the natural real interest rate as

$$r_t^* = c \cdot g_t + w_t$$

where g_t is annualized quarterly growth rate of potential output. For the second approach, the natural rate of interest is determined indirectly through the Taylor rule, as is done in Kirker (2008). The description of the data used in the estimations is given in Appendix A.

4.1 The Laubach-Williams model

In it's general form, the Laubach-Williams model is given by:

$$\tilde{y}_t = A_y(L)\tilde{y}_{t-1} + A_r(L)(r_{t-1} - r_{t-1}^*) + \epsilon_{y,t} \tag{8}$$
$$\pi_t = B_\pi(L)\pi_{t-1} + B_y(L)\tilde{y}_{t-1} + B_x(L)x_t + \epsilon_{\pi,t} \tag{9}$$
$$r_t^* = cg_t + w_t \tag{10}$$
$$y_t^* = y_{t-1}^* + \frac{g_{t-1}}{400} + \epsilon_{y^*,t} \tag{11}$$
$$g_t = g_{t-1} + \epsilon_{g,t} \tag{12}$$
$$w_t = w_{t-1} + \epsilon_{w,t} \tag{13}$$

where $\tilde{y}_t = 100(y_t - y_t^*)$, y_t is logarithm of output, y_t^* is logarithm of potential output, r_t is real interest rates, π_t is annualized quarterly inflation, and x_t is a vector of various relative prices introduced to account for exogenous shocks affecting the price level, such as the relative price of imports. Equations (8) and (9) are the IS and Phillips curves. We assume that potential output growth, g_t, and the aforementioned composite, w_t, follow a random walk.

4.1.1 Estimation and Results

Following Laubach and Williams, we set

$$A_y(L) = a_{y,1} + a_{y,2}L \tag{14}$$

$$A_r(L) = a_{r,1} + a_{r,2}L \tag{15}$$

$$B_\pi(L) = b_{\pi,1} + b_{\pi,2}(L^2 + L^3 + L^4) + b_{\pi,3}(L^5 + L^6 + L^7 + L^8) \tag{16}$$

$$B_y(L) = b_{y,1} \tag{17}$$

where L is the lag operator. We further require $b_{\pi,1} + b_{\pi,2} + b_{\pi,3} = 1$. We diverge slightly from the original paper by setting $B_x(L) = 0$. There are two reasons for this: firstly, $B_x(L) \neq 0$ would require the use of data not used in Kirker (2008), on which the other model in this section is based, which would complicate the comparison of the two models; secondly, we do so for sake of parsimony.

We rewrite the model in the appropriate state-space form and estimate it using the Kalman filter. Estimation based on different initial values for the state variables and coefficients, including diffuse priors, indicate that many local maxima exist. Moreover, small changes in initial values produced wildly different estimates, suggesting that the likelihood function is relatively flat. In addition, the covariance matrix frequently converges to a singular matrix. Since these issues are not reported in the literature, they may be related to the Icelandic data. In light of these estimation difficulties, initialization and output evaluation must be guided by theory and aided by practical nudging. To initialize, we create time series for the unobservables from means of their observable counterparts, and estimate a corresponding system with least squares to get initial guesses for the coefficients. We use the means of the observable counterparts as a guide to our initial guesses for the Kalman filter. The estimate of the equilibrium or the natural real interest rate is given in Figure 7.[6]

[6] The estimates of the other state variables are given in Appendix D.

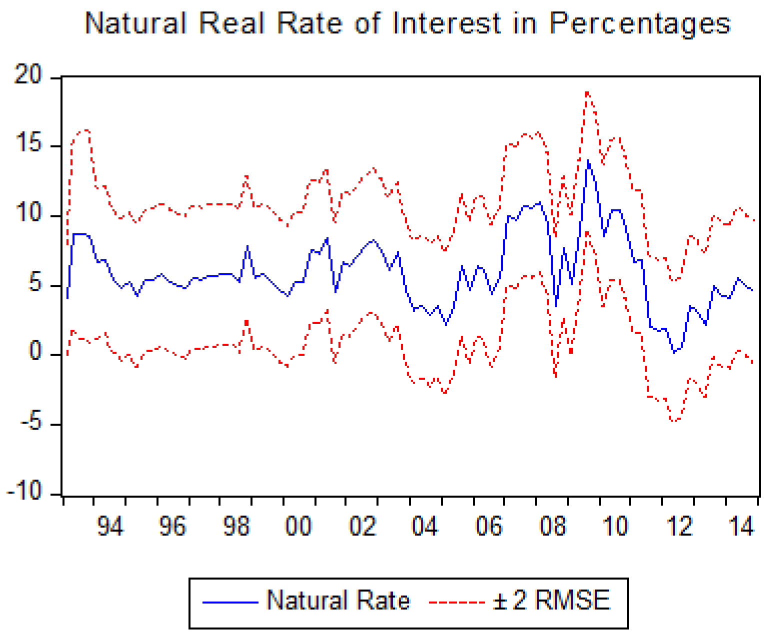

Figure 7. An estimate of the natural real interest rate in percentages via the Laubach-Williams Model

The estimate of the residual non-growth determinant of the natural interest rate, w_t, in Equation (10), is negative for roughly a decade, starting at the turn of the millennium. This is slightly discomforting as this value is intended to capture, and to be dominated by, the effects of time preference and risk premium, both of which should be non-negative. One interpretation of this result is that the coefficient c has not been constant over time, and has been smaller in recent years, implying that the effects of growth on the natural interest rate in recent times is overestimated by the Laubach-Williams model. As Taylor and Wieland (2016) argue, it might also be caused by misspecification due to omitting "... important variables relating to structural policy and monetary policy."

4.2 A New Keynesian Model for a Small Open Economy

Laubach and Williams (2003) estimate a model of a closed economy, which is not unreasonable for the US but less so for Iceland. In this section we broadly follow Kirker (2008) in our specification with a notable difference being that our model is backward-looking. Foreign variables are allowed to affect the domestic economy, both through the output determination and through the pseudo-UIP[7] condition, which reduces to the standard UIP condition if we lead all variables in the equation by 1 step and introduce expectations, and further require $d_{z,1} = -d_{z,2} = d_r = 1$. The model is given by:

$$\tilde{y}_t = A_y(L)\tilde{y}_{t-1} + A_r(L)(r_{t-1} - r^*_{t-1}) + a_z(z_{t-1} - z^*_{t-1}) + a_f(y_{f,t-1} - y^*_{t-1}) + \epsilon_{y,t} \tag{18}$$

$$\pi_t = B_\pi(L)\pi_{t-1} + B_y(L)\tilde{y}_{t-1} + B_x(L)x_t + \epsilon_{\pi,t} \tag{19}$$

$$r_t = \gamma_r r_{t-1} + (1-\gamma_r)\left(r^*_t + c_\pi(\pi^A_{t-1} - \pi^{A*}_{t-1}) + c_y\tilde{y}_{t-1}\right) + \epsilon_{r,t} \tag{20}$$

$$z_t = z^*_t + d_{z,1}z_{t-1} + d_{z,2}z^*_{t-1} + d_r((r_{t-1} - r^*_{t-1}) - (r_{f,t-1} - r^*_{f,t-1})) + \epsilon_{z,t} \tag{21}$$

$$r^*_t = r^*_{t-1} + \epsilon_{r^*,t} \tag{22}$$

$$y^*_t = y^*_{t-1} + \frac{g_{t-1}}{400} + \epsilon_{y^*,t} \tag{23}$$

$$g_t = g_{t-1} + \epsilon_{g,t} \tag{24}$$

$$z^*_t = z^*_{t-1} + \epsilon_{z^*,t} \tag{25}$$

$$\pi^*_t = \pi^*_{t-1} + \epsilon_{\pi^*,t} \tag{26}$$

where z^* is the logarithm of the equilibrium real exchange rate, and π^* is the inflation target, both of which are unobservable and assumed to follow a random walk. Variables with the subscript f are foreign variables. The variables π^A_t and π^{A*}_t are the four-quarter percentage change in the price level and price level target. The first two equations are the small open economy IS curve and Phillips curve. Equation (21) is the aforementioned pseudo-UIP condition. Lastly, all unobservables are assumed to evolve according to random walk processes, with the exception of potential output, the logarithmic difference of which is equal to potential output growth and a stochastic component (cf. Equation 23 and 24).

4.2.1 Estimation and Results

This model suffers from the same estimation problems when applied to the Icelandic data as the Laubach-Williams model, with the likelihood function being flat and having many local maxima. We apply the same methodology as we did when estimating the Laubach-Williams model in finding suitable guesses for the coefficients, and in finding starting values for the Kalman

[7]We use a pseudo-UIP condition in Equation (21) since, in general, the UIP condition hasn't been found to hold for the Icelandic economy.

filter. That is, we set up a corresponding system with proxies for the unobservable variables, and estimate the parameters with least squares to get initial guesses to initialize the Kalman filter, which is then used to estimate the model.

One implication of our estimated model is that inflation has been close to the central bank's inflation target throughout the estimation period, but consistently over it between 1997 and 2009. This can be seen by using the HP-filter on CPI inflation and the inflation target estimate[8]. The difference between the filtered time series is shown in Figure 8.

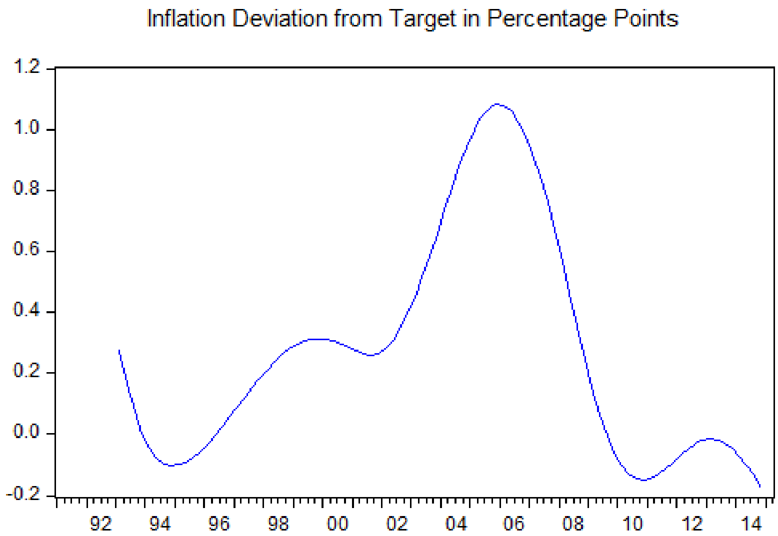

Figure 8. Difference in percentage points between the trend of quarterly annualized inflation, and the trend of the estimated implied inflation target, i.e. $\pi_t - \pi_t^*$

Our interest, however, lies mainly in the estimate of the natural real rate shown in Figure 9. Of note is the large uncertainty of the estimate, indicated by the two red dash lines in the figure. The average natural rate over the period is 3.1%, which is very close to our prior beliefs.

[8]We use the HP-filter since the estimated inflation target is unreasonably volatile, making it very difficult to gauge the stance of monetary policy without slight modifications.

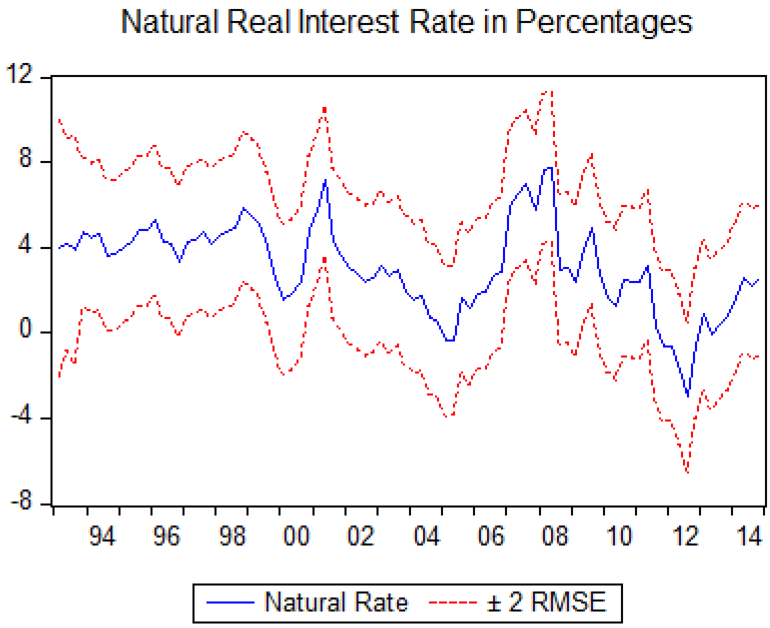

Figure 9. Estimate of the Natural Real Interest Rate in percentages via the New Keynesian Model for SOE

5 Empirical investigation using a forward-looking model

In this section the model's agents form their expectations based on both backward-looking (adaptive) considerations and rational expectations. The model at its core is akin to the estimated small, semi-structural, New Keynesian model for Iceland in Honjo and Hunt (2006). However, they do not model the natural rates which is our main interest in this exercise. In this respect we follow Kirker (2008), who estimates time-varying natural rates for New Zealand. We use the same adjustments and specifications since New Zealand and Iceland are small open-economies with similar characteristics. The model is adapted from Berg et al. (2006) and follows a standard two country framework where the domestic economy is assumed to be a small open economy, and the foreign economy represents the rest of the world. An exchange rate relationship between the economies is specified in real terms and implicitly assumes a complete pass-through as in Gali and Monacelli (2005).

A common method, often implemented in these kind of exercises, is to model the growth rate of potential output by assuming it to follow a local linear trend. However, as noted in Kirker (2008), since the model has been extended to model the natural rates, some of the parameters may be weakly identified. Therefore, the choice of priors becomes important in anchoring parameter values. One of these weakly identified parameters is the standard deviation of shocks to the growth rate of potential output. In an attempt to improve identification, we do not model the growth rate of potential output explicitly but treat the output gap as an observable. The output gap we use is the estimated output gap from the CBI's QMM-model (Daníelsson et al., 2015).

5.1 The Model

5.1.1 Domestic economy

Core domestic behavioural equations

The model has four core equations: an IS-curve, New Keynesian Phillips-curve, modified uncovered interest rate parity condition, and a Taylor-type monetary policy rule. We begin by describing aggregate demand by specifying an IS-relationship of the form:

$$\tilde{y}_t = (1-\beta_y)E_t\tilde{y}_{t+1} + \beta_y\tilde{y}_{t-1} - \beta_r\tilde{r}_{t-1} - \beta_z\tilde{z}_{t-1} + \beta_f\tilde{y}_t^f + \epsilon_t^y \qquad (27)$$

where current output gap, $\tilde{y}_t$, depends on last periods real activity, $\tilde{y}_{t-1}$, and next periods expected real activity, $E_t\tilde{y}_{t+1}$, as well as last period's real interest rate gap, $\tilde{r}_{t-1}$, last period's real exchange rate gap, $\tilde{z}_{t-1}$, and foreign demand, i.e foreign output gap, $\tilde{y}_t^f$

Inflation is modelled using a hybrid New Keynesian Phillips curve:

$$\pi_t = (1-\alpha_\pi)E_t\pi_{t+1} + \alpha_\pi\pi_{t-1} + \alpha_y\tilde{y}_{t-1} - \alpha_z(z_t - z_{t-1}) + \epsilon_t^\pi \qquad (28)$$

To anchor inflation to a stable level, the Central Bank is assumed to conduct monetary policy according to the following forward-looking Taylor-type rule:

$$i_t = \gamma_i i_{t-1} + (1-\gamma_i)\left[r_t^* + E_t\pi_{t+1}^* + \gamma_\pi E_t\left(\pi_{t+4}^A - \pi_{t+4}^{A,*}\right) + \gamma_y\tilde{y}_t\right] + \epsilon_t^i \quad (29)$$

where γ_i is a smoothing parameter in the level of the nominal interest rate, i_t. The Central Bank moves the nominal interest rate in response to deviations in expected annual inflation from its annual target, $E_t(\pi_{t+4}^A - \pi_{t+4}^{A,*})$, and the current output gap, $\tilde{y}_t$.

Identities

The annualized inflation target is assumed to follow a random walk process:

$$\pi_t^* = \pi_{t-1}^* + \epsilon_t^{\pi^*} \tag{30}$$

The ex-ante real interest rate is defined by the Fisher equation:

$$r_t = i_t - E_t \pi_{t+1} \tag{31}$$

The real interest rate gap is the difference between the real interest rate and its natural level:

$$\tilde{r} = r_t - r_t^* \tag{32}$$

where the natural real rate of interest is assumed to follow a random walk process:

$$r_t^* = r_{t-1}^* + \epsilon_t^{r*} \tag{33}$$

Annual inflation is given by:

$$\pi_t^A = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})/4 \tag{34}$$

and the annual inflation target:

$$\pi_t^{A,*} = (\pi_t^* + \pi_{t-1}^* + \pi_{t-2}^* + \pi_{t-3}^*)/4 \tag{35}$$

5.1.2 Exchange rate

The exchange rate is modelled using a modified uncovered interest rate parity condition, given by:

$$z_t = z_{t+1|t}^e + (r_t - r_t^f + \rho_t^*)/4 + \epsilon_t^z \tag{36}$$

where z_t is the logarithm of the real exchange rate, $z_{t+1|t}^e$ is the expected logarithm of next period's real exchange rate, ρ^* is the equilibrium risk premium and ϵ_t^z is a shock to the risk premium. Division by four is due to the real interest rate terms being expressed in annual terms.

Expectations of the real exchange rate are formed as a standardized[9] linear combination of the next period's rationally expected value, $E_t z_{t+1}$, and the previous period's observation, z_{t-1}:

$$z_{t+1|t}^e = \delta_z E_t z_{t+1} + (1 - \delta_z) z_{t-1} \tag{37}$$

[9]i.e. coefficients sum to unity

The equilibrium risk premium is defined as:

$$\rho_t^* = 4[z_t^* - \delta_z E_t z_{t+1}^* - (1-\delta_z) z_{t-1}^*] - r_t^* + r_t^{f*} \tag{38}$$

where z_t^* is the equilibrium level of the real exchange rate, r_t^* is the equilibrium real interest rate of the domestic economy and r_t^{f*} is the equilibrium real interest rate of the foreign economy. When expectations are fully rational, i.e. $\delta_z = 1$, Equation (36) becomes the standard UIP condition in gap form.

The real exchange rate gap is defined as the difference between its observed level and the equilibrium rate:

$$\tilde{z} = z_t - z_t^* \tag{39}$$

We assume that the equilibrium real exchange rate follows a random walk process:

$$z_t^* = z_{t-1}^* + \epsilon_t^{z*} \tag{40}$$

5.1.3 Foreign economy

The foreign economy is a detrended closed-economy version of the domestic economy[10].

IS-curve:

$$\tilde{y}_t^f = (1-\beta_y^f) E_t \tilde{y}_{t+1}^f + \beta_y^f \tilde{y}_{t-1}^f - \beta_r^f r_{t-1}^f + \epsilon_t^{y,f} \tag{41}$$

Hybrid New Keynesian Phillips curve:

$$\pi_t^f = (1-\alpha_\pi^f) E_t \pi_{t+1}^f + \alpha_\pi^f \pi_{t-1}^f + \alpha_y^f + \epsilon_t^{\pi,f} \tag{42}$$

Monetary policy rule:

$$i_t^f = \gamma_i^f i_{t-1}^f + (1-\gamma_i^f) \left(\gamma_\pi^f E_t \pi_{t+4}^{A,f} + \gamma_y^f \tilde{y}_t^f \right) + \epsilon_t^{i,f} \tag{43}$$

Foreign ex-ante real interest rate is given by the Fisher equation:

$$r_t^f = i_t^f - E_t \pi_{t+1}^f \tag{44}$$

And the annual inflation rate is:

$$\pi_t^{A,f} = (\pi_t^f + \pi_{t-1}^f + \pi_{t-2}^f + \pi_{t-3}^f)/4 \tag{45}$$

[10] The equilibrium of each foreign variable, such as the natural real rate of interest, is not modelled explicitly but calculated as the trend of the observables in the data.

5.2 Estimation and results

5.2.1 Priors

Our choice of prior distributions for some parameters are similar to the ones used by Kirker (2008). The choice of priors for weakly identified parameters are particularly important since it will govern the value of the estimated posterior distribution. We therefore try to find reasonable values for these priors through other studies where appropriate, for example from the estimated values in Honjo and Hunt (2006), who estimate a model for Iceland using data from 1992Q2 to 2005Q2, or simply with economic intuition. The choice of priors and the resulting posterior distributions are summarized in Table 4 and 5 below. Plots of the distributions are in Appendix B.

Identification analysis shows that some parameters are weakly identified. An indication of this can be seen in the figures of their prior distribution plotted against the estimated posterior distribution where the data seems to have little effect on the estimation of the posterior distribution of some parameters[11].

We choose a rather low prior mean of 0.01 for the sensitivity of the domestic economy to the real exchange rate gap, β_z. This is the same prior as used in Kirker (2008) and almost the same as the estimated posterior mean in Honjo and Hunt (2006). The prior for the sensitivity of the domestic economy to foreign demand, β_f, is also chosen to be rather low and is close to the estimated value in Kirker (2008).

Some of the standard deviation of shocks are not well identified. When using a relatively low value for the prior for the standard deviation of shocks to the inflation target, $\sigma^{\epsilon}_{\pi^*}$, as is done in Kirker (2008), we ran into some problems which resulted in a fairly constant inflation target throughout the sample, with values close to the average inflation over the sample. To get a more believable estimate, with more variation in the target, we had to raise that prior to 0.5.

We follow Kirker (2008) and set the same prior for the standard deviation of shocks to the natural real interest rate, $\sigma^{\epsilon}_{r^*}$, since there is little information available from other studies specific to the Icelandic economy on the appropriate distribution and mean for this particular prior.

[11]Although similarities between prior and posterior distributions is evidence of weak identification, it is not sufficient since it is possible that the prior is supported by the data.

In Honjo and Hunt (2006) the estimated posterior mean of the standard deviation of real exchange rate shocks, σ_z^ϵ, is 2.06, which is very close to the prior chosen by Kirker (2008). We therefore set the prior mean to 2.0, and choose the same relative value for the standard deviation of shocks to the equilibrium real exchange rate, $\sigma_{z^*}^\epsilon$, which is half the mean of the prior for σ_z^ϵ. It is our prior belief that shocks to the real exchange rate are significantly larger than the shocks to the equilibrium real exchange rate.

5.2.2 Posteriors

Table 4 and 5 present the results of the estimation, the means and confidence intervals of the posterior distributions[12]. The results show a relatively low persistence in the IS-curve, with the weight on lagged output gap estimated at $\beta_y = 0.2760$. This is roughly half of the estimated value in Honjo and Hunt (2006). We can also see that the domestic output gap is rather insensitive to the real exchange rate gap, as was expected, because this is a result of the choice of prior for that particular parameter. If we look at the plot of its posterior distribution in Appendix B we can see that its value is mainly driven by the chosen prior distribution. The same applies to the sensitivity to foreign demand, $\beta_f = 0.0494$. This shows the weak identification of these parameters in the model, as we discussed in the preceding section.

Contrary to our prior belief, the standard deviation of shocks to the equilibriuml exchange rate, $\sigma_{z^*}^\epsilon = 4.34\%$, is much larger than the standard deviation of the real exchange rate shocks, $\sigma_z^\epsilon = 0.92\%$. This is, however, similar to the result in Kirker (2008) who finds the standard deviations to be 1.319% for the equilibrium exchange rate, and 1.066% for the real exchange rate shocks. The parameter $\sigma_{z^*}^\epsilon$ is well identified in the model, but σ_z^ϵ is not, as can be seen in the plot of the posterior distribution in Appendix B, which appears bimodal.

As mentioned before, first run of the model gave us a fairly constant inflation target with little variation around a value equal to the average inflation over the sample. We therefore increased the mean of the prior for the standard deviations of shocks to the inflation target to get more variation. This resulted in an estimated posterior mean of 0.78%. This value is considerably larger than what Kirker (2008) finds, but that is not necessarily a surprise considering the data sample we use which includes high inflationary periods,

[12] See Appendix B for plots of the posterior distributions.

especially during the financial crisis in 2008-2009, and the fact that the CBI didn't officially use an inflation target until 2001.

Table 4. Results from Metropolis-Hastings (parameters)

Parameter	Prior dist	Prior mean	Prior s.d.	Posterior mean	HPD inf	HPD sup
β_y	β	0.6	0.15	0.2760	0.1903	0.3603
β_r	Γ	0.1	0.05	0.0346	0.0104	0.0586
β_z	Γ	0.01	0.005	0.0102	0.0035	0.0171
β_f	Γ	0.05	0.02	0.0483	0.0194	0.0755
α_π	β	0.5	0.2	0.3502	0.2628	0.4374
α_y	Γ	0.1	0.05	0.0324	0.0125	0.0505
α_z	Γ	0.075	0.05	0.0754	0.0211	0.1271
γ_i	Γ	0.7	0.2	0.8066	0.7551	0.8590
γ_π	Γ	2.0	0.5	2.5099	1.6620	3.3654
γ_y	Γ	1.0	0.3	0.6213	0.3304	0.9069
δ_z	Γ	0.75	0.2	0.2674	0.1851	0.3522
β^f_y	β	0.4	0.2	0.7346	0.6130	0.8561
β^f_r	Γ	0.1	0.1	0.1511	0.0824	0.2193
α^f_π	β	0.5	0.2	0.1533	0.0757	0.2302
α^f_y	Γ	0.1	0.05	0.0748	0.0462	0.1030
γ^f_i	β	0.7	0.25	0.7950	0.7162	0.8720
γ^f_π	Γ	1.75	0.75	2.0789	1.0001	3.0700
γ^f_y	Γ	1.0	0.5	0.7808	0.3909	1.1583

Table 5. Results from Metropolis-Hastings (standard deviation of structural shocks)

Parameter	Prior dist	Prior mean	Prior s.d.	Posterior mean	HPD inf	HPD sup
ϵ_y	Γ^{-1}	0.5	Inf	1.4276	1.2208	1.6291
ϵ_π	Γ^{-1}	0.5	Inf	1.8083	1.5472	2.0661
ϵ_i	Γ^{-1}	0.5	Inf	0.7573	0.6200	0.9006
ϵ^*_π	Γ^{-1}	0.5	Inf	0.7807	0.1476	1.3762
ϵ^*_r	Γ^{-1}	0.2	Inf	0.6166	0.0614	1.0889
ϵ_z	Γ^{-1}	2.0	Inf	0.9161	0.4903	1.3436

(Continued on next page)

Table 5. (continued)

Parameter	Prior dist	Prior mean	Prior s.d.	Posterior mean	HPD inf	HPD sup
ϵ_z^*	Γ^{-1}	1.0	Inf	4.3403	3.7436	4.9300
ϵ_y^f	Γ^{-1}	0.5	Inf	0.2727	0.2217	0.3224
ϵ_π^f	Γ^{-1}	0.5	Inf	0.7297	0.6249	0.8329
ϵ_i^f	Γ^{-1}	0.5	Inf	0.2944	0.2538	0.3352

5.2.3 Natural rates

The path of the unobservable natural rates within the model is extracted using the Kalman smoother. The estimated natural real rate of interest can be seen in Figure 10 along with the ex-ante real interest rate within the model (cf. Equation (31) above). The estimated natural rate at the end of 2015 is 3.6%. It has increased since its lowest point during the first half of 2011, but the increases during the last 2-3 years have been small. The average of the real interest rate is about the same as the natural rate in 2015.

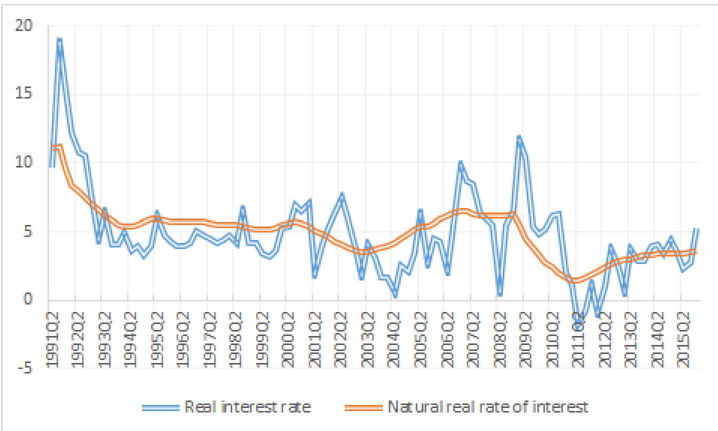

Figure 10. Natural real rate of interest and the ex-ante real interest rate within the model, 1991Q2-2015Q4.

Monetary policy stance

As an indicator of the stance of monetary policy we can look at the real interest rate gap $(r_t - r_t^*)$ in Figure 11.

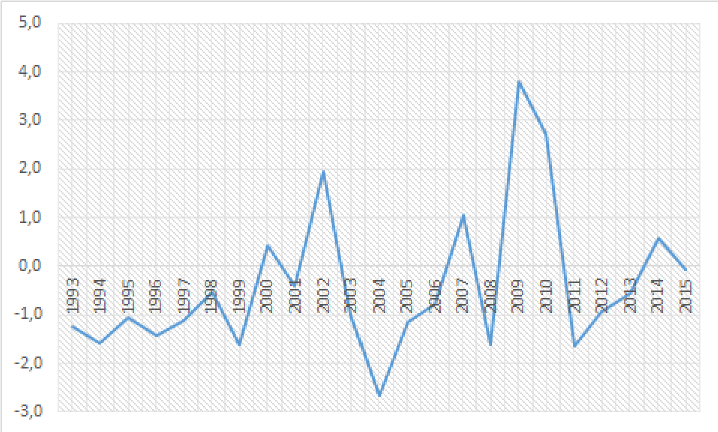

Figure 11. The stance of monetary policy within the model: $r_t - r_t^*$

The period from 1993 to 1999 was fairly expansionary with an average real interest rate gap of -1.2 percentage points. In 2000 and 2001 the stance of monetary policy is approximately neutral. After that, in 2002, the stance becomes contractionary where the gap reaches +1.9 percentage points. In the years preceding the crisis, 2003 to 2006, we can see that the stance of monetary policy is expansionary, with an average gap of -1.4 percentage points, and 2004 being the most expansionary year with a gap of -2.7 percentage points. In 2007 the stance turns contractionary with a gap of +1.1 percentage point in 2008, when the financial crisis hit the Icelandic economy, the stance is expansionary with a gap of -1.6 percentage points. In 2009, soon after the crisis hit Iceland, the real interest rate rises fast above the natural real rate with a gap of 3.8 percentage points, and 2.7 percentage points in 2010. This was a period of a very high inflation. In 2011-2013 the gap becomes negative for the whole period, with an average of -1.0 percentage points. From the year 2014 the stance is contractionary until it becomes neutral in 2015.

Inflation and target

Inflation over the period 1991Q2 to 2015Q4, and the model estimate of the inflation target can be seen in figure 12. The average annual inflation over the whole sample is 4.3%, and the average of the estimated annual target is 4.5%. The average of the estimated target from 2011, and until the end of 2015 is around 3.5%. These estimates are somewhat higher than the official target of the CBI which has been 2.5% since 2001. However, since 2009 it has been moving towards the official target, and the estimate for 2015 is 2.8%

(see Figure 12). Before 2001 the official target variable of the bank was the exchange rate.

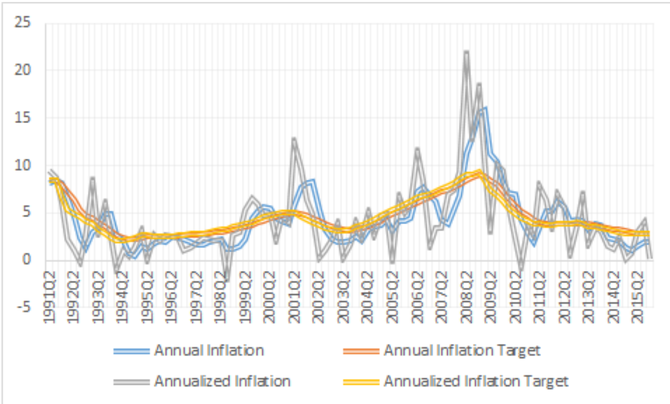

Figure 12. Inflation and target within the model, 1991Q1-2015Q4

6 Comparison of the Empirical Estimates

Estimates of potential output from both backward-looking models in Section 4 are very similar, as can be seen from Figure 13, which shows that the output gap estimates from the two models are very close.

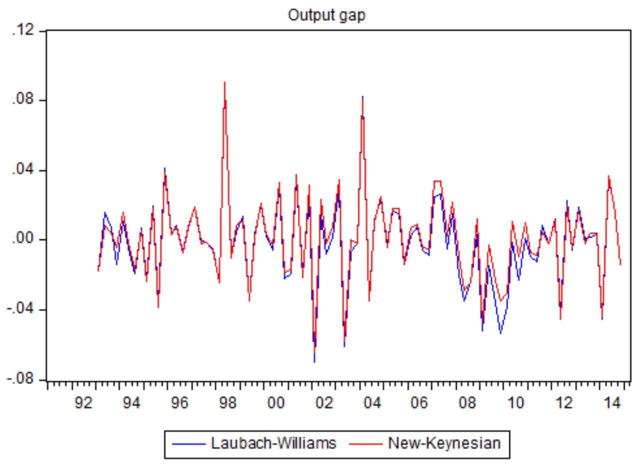

Figure 13. Percentage deviation of the estimated potential output from actual output for the two different models considered.

However, the estimated natural real interest rate from the Laubach-Williams model is consistently higher than the New Keynesian open economy model estimate, while the dynamics of the two estimates are similar for most of the period. Furthermore, the Laubach-Williams model gives an estimate which is persistently higher than our prior beliefs about the natural interest rate, and is very far from the estimates of other countries. The estimated natural real interest rate using the two methods and the CBI policy rate are shown in Figure 14.

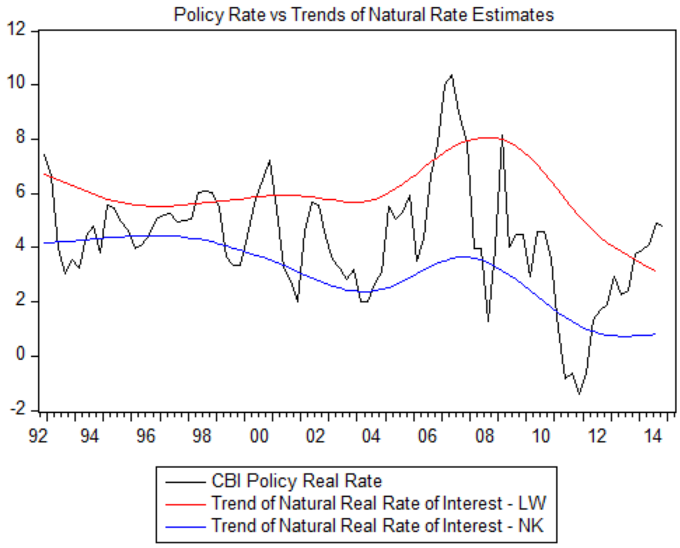

Figure 14. The CBI Policy Real Rate is the ex-ante average policy rate over the quarter

Economic intuition tells us that the monetary stance, i.e. $r_t - r_t^*$, cannot be consistently contractionary over the boom-bust cycle. But this is indeed the case according to the backward-looking New Keynesian model and can be seen from Figure 14, indicating that the Laubach-Williams provides more plausible estimates. However, as aforementioned, the Laubach-Williams estimate is considerably over our prior beliefs for the natural interest rate. The forward-looking estimates described in section 5.2.2, both passes the sniff test of having plausible dynamics in the boom-bust cycles and conforming to our prior beliefs. Such coarse assessment of the plausibility of the natural rate estimates indicates that the forward-looking estimates are more accurate.

A comparison of all three empirical estimates of the natural real interest rate can be seen in Figures 15 and 16.

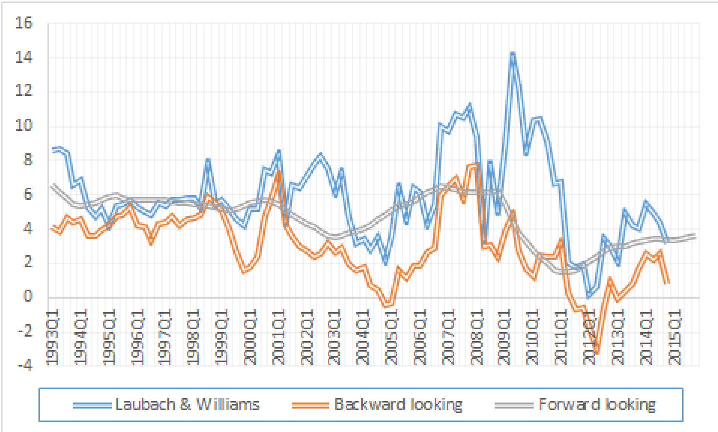

Figure 15. Comparison of model estimates of the natural rate of interest.

Figure 16. Yearly average of the natural rate of interest from the backward- and forward- looking models.

Firstly, the forward-looking estimate seems to fall roughly between the two backward-looking estimates, and secondly, the average of the two backward-looking estimates appear to evolve in time similarly to the forward-looking estimates with few quarters lag. The similarity is evidence of the models' ability to infer the natural rate of interest from the data. Further, it diminishes the risk of having to confront the pile-up problem[13] described by Stock and Watson (1996), and associated with the problem of estimating the

[13]The pile-up problem refers to the difficulty of estimating state-space models with MLE

natural interest rate via frequentist inference.

The models, in unison, indicate that the natural rate of interest has fallen in response to the downturn in 2008, in the wake of the collapse of the financial system, but has risen again in recent years and seems to be stabilizing between 3% and 4%, although it is perhaps premature to conclude that stabilization has been achieved. A similar fall in the equilibrium rate can be seen between 2001 to 2004, after the economy suffered a recession. The dynamics of the natural interest rate between 2001 and 2008 do, however, not necessarily stem from the same source as the 2009-2015 dynamics since the Icelandic economy went through a few important changes in the former period, including privatisation of two out of the three large commercial banks, floating of the exchange rate, independence of the Central Bank and the implementation of inflation targeting monetary policy. Recent studies[14] on natural real interest rates globally suggest that they have been on a downward trajectory since the 2008 downturn, which might influence the Icelandic rates through arbitrage opportunities and carry trade. In their paper, Holston et al. (2016), find that output and interest rates in Canada, Euro Area and the UK have been falling, and in addition that there is "a substantial amount of comovement over time, suggesting an important role for global factors in shaping trend growth and natural rates of interest." However, capital controls in Iceland were put in place in 2008 which should decrease such opportunities and reduce the downward pressure on the natural interest rate from abroad. Moreover, a structural explanation can be given in terms of growth of potential output, predicated on a positive relationship between potential output, and the natural interest rate, as is implied by the Euler equation, and employed by Laubach and Williams (2015); if potential output growth has decreased over the period, or the influence of the growth of potential output on the natural interest rate has decreased[15], one would expect the natural interest rate to fall, ceteris paribus.

It is not straightforward to determine which estimate of the natural real rate of interest is best. In such cases it makes sense to use a weighted average of the different estimates. Taking into account the results of Kim and Kim (2013), who suggest that the pile-up problem is much less severe and the estimation more robust with Bayesian methods, as well as the aforemen-

when the signal-to-noise ratio for the non-trend component of the unobservable variable is low.

[14]See e.g. Lubik and Matthes (2015), Laubach and Williams (2015)

[15]Which implies a change in the intertemporal elasticity of substitution, as is indicated in the estimation of the Laubach-Williams model

tioned arguments for the forward-looking estimate being more plausible, the forward-looking model has double the weight of the two backward-looking ones[16]. The result can be seen in Figure 17. We can see in the figure that after a 5-year period of decreasing natural real rate, from the peak in 2007 to the lowest level in 2012, our estimate is trending upwards, reaching 3.3% in 2014 and 3.5% in 2015.

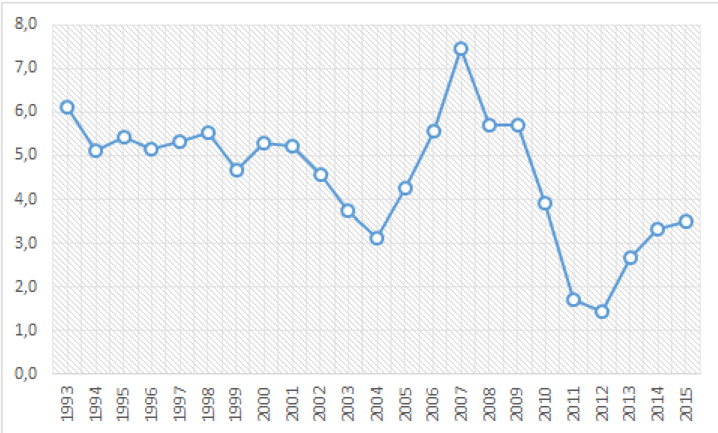

Figure 17. Weighted average of the estimated natural real rate of interest from each model.

Despite the reassuring congruence in the estimates, noted above, they are all different, especially in their mean. In addition, there is sizeable uncertainty of the estimates due to measurement errors, possible misspecification and the pile-up problem. The estimates provided are, however, valuable evidence, especially on the dynamics of the natural interest rate over time.

7 Discussion

In this paper we have applied different methods that are commonly used for estimating the equilibrium real rate of interest to Icelandic data. In Section 2 we used the marginal productivity of capital to estimate the natural rate. The estimates show a clear negative trend in this rate since 1994 and a sharp decline during the financial crisis. The main reason for the long run negative trend is an upward trend in the share of wage cost in total factor income in Iceland. The estimated value for the business sector in 2014 is close to

[16]In 2015 the forward-looking estimate has the weight 1.

10%. This is an upper bound for the actual value as the estimates in this section ignore risk premiums and also the value of land, natural resources and intangible capital, e.g. human capital, which are important parts of the stock of capital in Iceland. The effects of these factors should be subtracted from the above estimate of 10%.

Section 3 discusses estimations based on the Euler equation. The estimates vary a lot depending on assumptions concerning the utility function and the discount rate. The estimates presented in Tables 1 and 2 in Section 3 above are based on the discount rate in DYNIMO, CBI's DSGE-model, which is 3.5%. If the discount rate is assumed lower, the estimated equilibrium real rate is equally lower and some of the estimates in Table 1 and 2 become closer to the estimates in Sections 4 and 5. There are some indications that private consumptions in Iceland behaves differently from what it did before the financial crises of 2008. Possibly the impatience of Icelandic consumers has decreased and is reflected in a lower discount rate.

Assuming logarithmic utility, which implies relative risk aversion of unity, and using linear approximation of the Euler equation, assumptions that have been widely used in recent years, gives an estimate of the natural real rate of 5%. This should be regarded as an estimate of the long term equilibrium rate. Assuming utility functions with risk-aversion above unity leads to higher estimates. Even with moderate risk-aversion the estimates becomes unreasonably high. Adding a second, non-linear term, to the approximation of the Euler equation gives a natural rate of 4.7% if the coefficient of relative risk aversion is unity and 5.8% if it is 2. This method gives more reasonable estimates also when we allow utility functions which exhibit higher and possibly more reasonable, risk-aversion. But including habit persistence into the utility functions as in DYNIMO, gives unreasonable and often negative values if the second, non-linear, term is included in the approximation of the Euler equation.

The methods used in Sections 4 and 5 provide time-varying estimates of the natural rate. The estimates exhibit some negative trend since the 1990s, a decline during the financial crisis and some recovery since 2011. The first model in Section 4, the celebrated model by Laubach and Williams gives an estimate of the natural rate of 5% by the end of 2014. The estimates have been rising from a low of almost 0% in 2012. The second method used in Section 4 gives somewhat lower estimates of the natural real rate, or 3% by the end of 2014 and has been rising from negative values in 2012. The method used in Section 5 gives an estimate of the natural rate of some 3.5% in 2015 and has been rising from 1.5% in 2010. The reason for this increase in the natural rate in recent years is the increase in the rate of growth in Iceland. The Laubach Williams model postulates a linear relationship between the growth of the potential production in the economy and the natural rate and the estimation gives that the relationship is positive. In the other models similar relationship is estimated without being so explicitly specified as it is in the Laubach Williams model. The changes over time in the different estimates are similar, and also similar to the changes over time in the estimates in Section 2, but the levels are somewhat different.

The models used in the estimation of the equilibrium real rates are extremely simplified versions of the complex reality of modern economies with large financial systems. In Section 2 it is assumed that the very different technical structures of the individual firms can be aggregated into a simple aggregate production function, and that the equilibrium condition in the production can be summarized by the aggregate marginal productivity of capital being equal to one single measure of the cost of capital. Modern financial markets offer different financial products, and different rates of interest depending on the associated risk and the cost of financial intermediation. Excluding these features from our models might lead to misspecification errors. Experience (especially from the early days of econometric modelling in 1970s and 1980s) shows though, that more complicated models rarely offer improved insights, or better forecasts. In fact, as Már Guðmundsson (2016), governor of the CBI, explained in a recent paper "[m]aking extreme assumptions is of paramount importance for economic theory-making." He also stressed the important caveat, that "this must be borne in mind when applying the theory to actual economic conditions."

In a recent OECD study of forecasting during the financial crises it is noted that the "main lesson of this research is that relying on more data does not mechanically improve forecast performance. This is because some economic indicators are redundant and because more data also means more noise to filter out."[17]

The simplifications used in this paper are similar to the ones used by researchers in other countries. In fact, we are mostly using models that have been used to analyse the same questions in other countries, and estimate their parameters, and "unobservables", using Icelandic data. We have included es-

[17] see https://oecdecoscope.wordpress.com/2016/10/18/forecasting-gdp-during-and-after-the-great-recession/

timation of the celebrated Laubach-Williams model where the economy is a closed one, and there is no exchange rate and therefore monetary policy cannot affect the rate of inflation through the exchange rate. It seems obvious that assuming a closed economy is less realistic in the case of Iceland than in the case of USA or the Euro Area where policy makers have made use of estimates from the Laubach-Williams model. Some researchers would though claim that models of USA, or the Euro Area, as closed economies, are ignoring important features of these economies. The estimation of the Laubach-Williams model using Icelandic data, gives rather high values for the natural real rate in Iceland, higher than the other state-space models. It is, however, reasonable to consider the results from this model. And perhaps the fact that the estimate of the natural real rate of interest based on a closed economy model is higher than the estimates based on open economy models tells us something about the Icelandic economy.

Most of the models in this paper focus on internal equilibrium conditions and ignore external equilibrium conditions. One model in Section 4, and the model in Section 5, contain foreign variables and model the exchange rate so that the equilibrium real rate of interest in the home country can be different from the real rate of interest abroad. An explicit equilibrium risk premium is used to explain that the domestic interest rate can be different from the foreign interest rate in equilibrium.

The statistical model in Section 5 does not take into account the various conditions that have affected the risk premium on Icelandic securities. The data span a period from the first half of the 1990s when Iceland still had capital controls and the period after the financial crisis in 2008 when capital controls were reimposed. It also ignores that the financial market in Iceland are much better known, and has stronger links with foreign financial markets, than they had some 20 years ago. This may lead to overestimation of the present risk premium on Icelandic securities in the absence of all capital controls. There are also indications that the risk premium may be quite sensitive to political factors that are difficult to quantify.

We think that the paper presents some reasonable estimates of the natural real rate of interest in Iceland. The standard deviation of the error in these estimates is though such that the size of a 95% confidence interval for them is several percentage points. It should also be borne in mind that the natural rate that matters for monetary policy is the rate during the next 1-2 years.

Many economists and policy makers have commented on the uncertainty around statistical estimates of the natural rate. In a speech given by S. Fischer, vice-chairman of the Federal Reserve, on October 17, 2016[18], he stressed that "there is a great deal of statistical uncertainty around all of these estimates", including the estimates by Laubach and Williams (2003). Bernhardsen and Kloster (2010) cite Blinder (1998) where he says about the natural rate that it is "difficult to estimate and impossible to know with precision. It is therefore most usefully thought of as a concept rather than as a number, as a way of thinking about monetary policy rather than as the basis for a mechanical rule..." Noting that the "normal rate is not directly observable in the market, and calculations are uncertain", Bernhardsen and Kloster (2010) comment that "Central banks must nevertheless have a perception of it in their rate-setting. Norges Bank has previously estimated that the normal real three-month money market rate has been in the interval of $2.5\% - 3.5\%$. With an inflation target of 2.5%, the normal nominal interest rate has then been in the interval of $5\% - 6\%$. In recent years, a normal real interest rate of 2.5% per cent has been assumed."

The variations in the interest rates and in the rates of inflation are much smaller in Norway than in Iceland, making estimated confidence intervals for the estimated natural rate larger in Iceland than in Norway. It may though still be sensible for the CBI to announce what it perceives as the natural rate of interest in Iceland even if it cannot back it up with precise statistical estimations. Perhaps it would make sense for the CBI to announce that the natural rate lies in some range rather than specifying a single number. It should though be noted that the models used in analysis and in forecasting at the CBI must be fed with precise numbers.[19] Larger variations in financial market conditions in Iceland compared to other countries should lead to more frequent reviews of the natural rate of interest, and the CBI should be expected to revise its assessment of the natural rate of interest more frequently than Central Banks in other countries.

[18] see https://www.federalreserve.gov/newsevents/speech/fischer20161017a.htm

[19] CBI has for many years published historical data used in its macro model, QMM, including the equilibrium real rate (RRN). It has been 3% since 2008Q4 when it was lowered from 4.5%. These data can be accessed at http://www.cb.is/monetary-policy/central-bank-of-iceland-economic-forecasts/.

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Appendices

A Data

We estimate the models using quarterly data for Iceland from 1991Q1 to 2014Q4 for the backward-looking models and to 2015Q4 for the forward-looking model. All data are taken from the QMM-database which is available at the CBI's website[20]. Detailed descriptions of the variables are available in the handbook for QMM which is located on the same link. For nominal interest rates we use the quarterly averages of end-of-month values of the interest rates of 7-day repurchase agreements (14-day before 2004, i.e. the variable RS in QMM). In the backward-looking models, the ex-ante interest rate, r_t, is computed as the nominal interest rate less the four quarter moving average of the CPI inflation, which is used as a proxy for expected inflation[21]. Inflation π_t is the percentage change in the CPI, which is compiled by Statistics Iceland (CPI). Output, y_t, is matched with logarithm of real GDP published by Statistics Iceland (GDP). The data for the real exchange rate, z_t, is the logarithm of the inverse of the real exchange rate published by the Central Bank of Iceland (REX), where the nominal exchange rate is trade weighted and the relative prices used in the calculations are the same as those that we use for domestic and foreign prices. All foreign variables are trade weighted indices constructed by the Central Bank of Iceland with respect to Iceland's main trading partners, with foreign prices as a percentage change of CPI (WCPI), foreign interest rates the interest rates on 3 month Treasury Bills (WRS), and foreign output is real GDP level (WGDP).

[20]http://www.cb.is/default.aspx?pageid=7fd288b8-0460-11e5-93fa-005056bc0bdb

[21]Laubach and Williams (2003) report that using the moving average as a proxy for expected inflation instead of a forecast from a univariate AR(3) model did not have a notable impact on the estimation.

B Posterior distributions

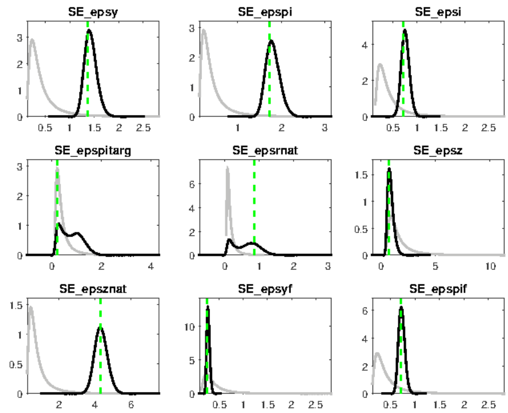


Figure 18. Prior and posterior distributions.

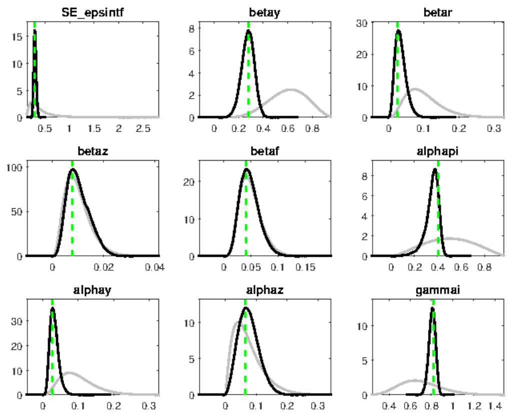


Figure 19. Prior and posterior distributions.

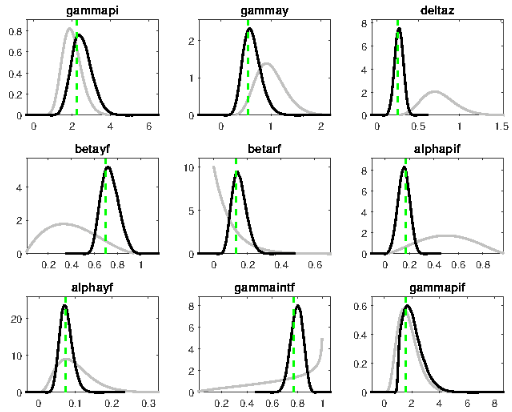


Figure 20. Prior and posterior distributions.

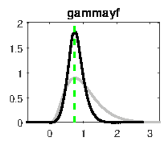


Figure 21. Prior and posterior distributions.

C Convergence diagnostics

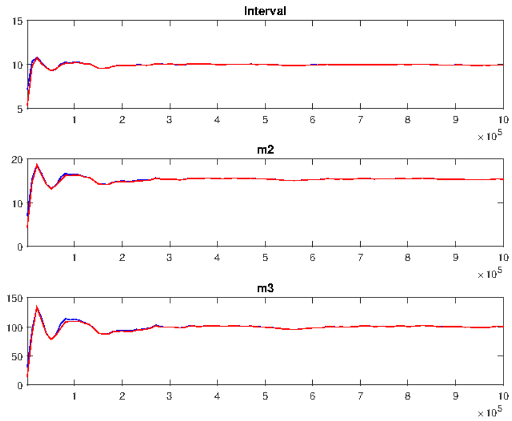
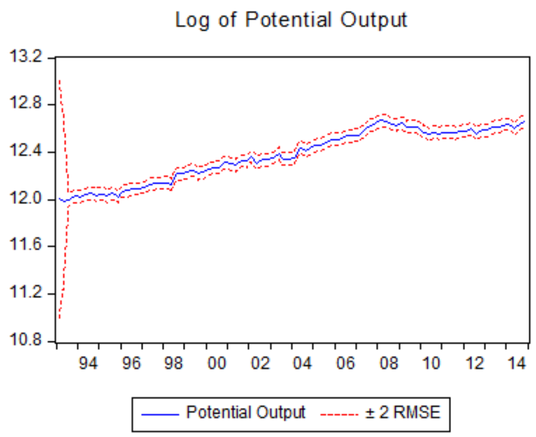
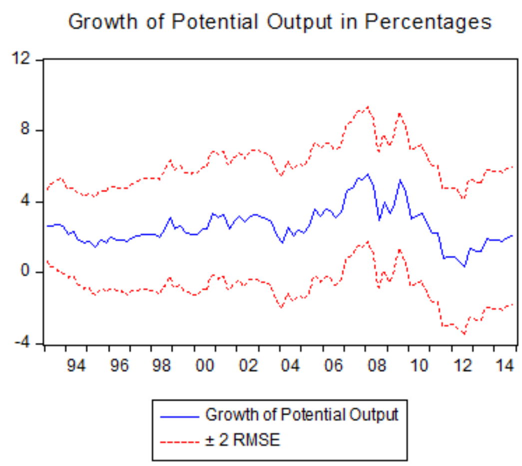


Figure 22. MCMC Multivariate convergence

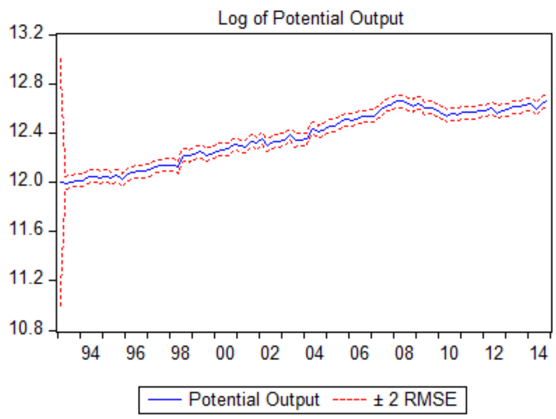
D Figures from estimates of backward-looking models

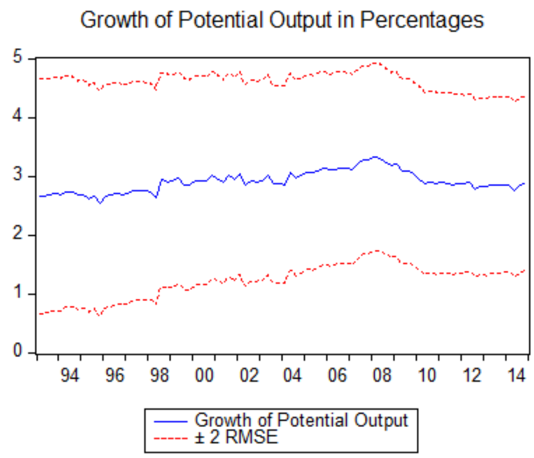
D.1 figures of Laubach-Williams model

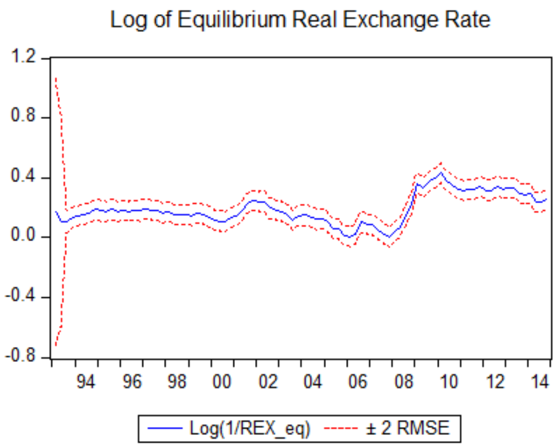




D.2 Figures of the NK model for a SOE






Log of Equilibrium Real Exchange Rate
1.2
0.8
0.4
0.0
-0.4
-0.8
94
96
98
00
02
04
06
08
10
12
14
Log(1/REX_eq)
± 2 RMSE